Unit 4: Statistics

Measures of Central Tendency &

Measures of Dispersion
Measures of Central Tendency

• a measure that tells us where the middle of a bunch of data lies

• most common are Mean, Median, and Mode.

Mean

• The Mean is the *average* of the numbers or a calculated "central" value of a set of numbers.

• To calculate: Add up all the numbers, then divide by how many numbers there are.

• Example: find the Mean \{3, 7, 5, 13, 20, 23, 39, 23, 40, 23, 14, 12, 56, 23, 29\}

  > The sum = 330

  > There are fifteen numbers

  > Mean = 330 / 15 = 22

• Example: find the Mean \{3, -7, 5, 13, -2\}

  > The sum = 3 - 7 + 5 + 13 - 2 = 12

  > There are 5 numbers

  > Mean = 12 ÷ 5 = 2.4
**Median**

- The *middle* number (in a sorted list of numbers).

- To find: place the numbers you are given in numerical order and find the middle number.
  > Example: find the Median of \{13, 23, 11, 16, 15, 10, 26\}
    - First, order the numbers, least to greatest
    - Next, identify the middle number

- What if there isn't only ONE middle number?
  - If there are an odd number of items in the list, there will be ONE definite Median or middle number.

  - If there are an even number of items in the list, there will be TWO middle numbers.
    > You will add those two numbers and divide by two to determine the median in this case.
    > Example:
      - find the Median \{3, 13, 7, 5, 21, 23, 23, 40, 23, 14, 12, 56, 23, 29\}
      - First, order the numbers, least to greatest
      \{3, 5, 7, 12, 13, 14, 21, 23, 23, 23, 23, 29, 40, 56\}
      - Next, identify the middle numbers

      **There are now fourteen numbers and so we don't have just one middle number, we have a pair of middle numbers:**

      \[3 \ 5 \ 7 \ 12 \ 13 \ 14 \ 21 \ 23 \ 23 \ 23 \ 23 \ 29 \ 40 \ 56\]

      - Finally, average the two numbers

      **Add them together and divide by 2:**

      \[21 + 23 = 44\]
      \[
      44 \div 2 = 22
      \]

      Median in this example is 22
**Mode**

- The number which appears **most often** in a set of numbers.

> Example: in \{6, 3, 9, 6, 6, 5, 9, 3\} the Mode is 6 (it occurs most often).

![](image.png)

**What if there is more than one mode?**

- You CAN have more than one mode.
  > Example: \{1, 3, 3, 3, 4, 4, 6, 6, 6, 9\}
    - 3 appears three times, as does 6.
    - So there are two modes: at 3 and 6

> Having two modes is called "bimodal".

> Having more than two modes is called "multimodal".

**What if there is NO mode?**

- You CAN have a set without a mode.
  > Example: \{1, 3, 5, 7, 9\}
    - No number appears more than once.
    - So there is NO mode.
Try the following:

Find the Mean, Median, and Mode for the following:

1) \{2, 3, 5, 6, 13, 5, 1\}

2) \{201, 199, 201, 200, 199\}

3) \{8, 7, 5, 19\}

4) \{3, 7, 21, 23, 63, 27, 29, 95, 23\}
Measures of Dispersion

- Tell you how widely spread out the values are
- Most common are Range, Standard Deviation, and Variance

Range

- The range of a data set is the difference between the maximum and minimum values in the set

Example:

> In \{4, 6, 9, 3, 7\} the lowest value is 3, and the highest is 9.

\[ \text{Range} = 9 - 3 = 6 \]

Example: In \{4, 6, 9, 3, 7\} the lowest value is 3, and the highest is 9. The single value of 3616 makes the range large, but most values are around 10.

So the range is \(9 - 3 = 6\).
Try the following:

Find the Range for the following:

1) \{2, 3, 5, 6, 13, 5, 1\} \hspace{1cm} 12

2) \{201, 199, 201, 200, 199\} \hspace{1cm} 2

3) \{8, 7, 5, 19\} \hspace{1cm} 14

4) \{3, 7, 21, 23, 63, 27, 29, 95, 23\} \hspace{1cm} 92
Variance

- The average of the **squared** differences from the Mean (symbol is $\sigma^2$)

- To calculate the variance follow these steps:
  > find the Mean (average of the numbers)
  > then for each number: subtract the Mean and square the result (the squared difference)
  > then work out the average of those squared differences

- Example: find the variance \{600, 470, 170, 430, 300\}
  > Step 1: Find the Mean

\[
\frac{600 + 470 + 170 + 430 + 300}{5} = \frac{1970}{5} = 394
\]

> Step 2: Subtract the Mean from each number in the set and square it

- $600 - 394 = 206$, $206^2 = 42,436$
- $470 - 394 = 76$, $76^2 = 5,776$
- $170 - 394 = -224$, $(-224)^2 = 50,176$
- $430 - 394 = 36$, $36^2 = 1,296$
- $300 - 394 = -94$, $(-94)^2 = 8,836$

> Step 3: Average those numbers (find the Mean) of the squared number

\[
\frac{42,436 + 5,776 + 50,176 + 1,296 + 8,836}{5} = \frac{108,520}{5} = 21,704
\]

> The variance, $\sigma^2 = 21,704$

$\sigma^2$ : sigma squared
Standard Deviation

- The Standard Deviation is a measure of how spread out numbers are (what is the standard difference between values in the set?)

- Its symbol is $\sigma$ (the greek letter sigma)

- The formula
  $\sqrt{\sigma^2} = \sigma$

- From our last example:
  > The variance, $\sigma^2 = 21,704$
  > The standard deviation, $\sigma = \sqrt{21,704} = 147.3227749 \approx 147.3228$
Variance/Standard Deviation:
small - data tends to be close to the mean
big - data is more spread out
Variance seems like ALOT of work!

GOOD NEWS...

It can ALL be done in your calculator!

Here's how: (I'm using example 4 from Try This)

> Go to STAT (enter to pull up lists)
  > enter the values in L1
    « order doesn't matter

> Once values are entered, 2nd Mode to go back to home screen.
> Go to STAT
> Arrow over to CALC
> Choose 1: 1-Var-Stats
  > hit enter 3 or 4 times to get the screen on the right

> Scroll down and there's more!
> All of your stats are here!

\[ X \text{ : mean} \]
What do they mean??

Mean

\( \overline{x} \)

Standard Deviation

this is the number of values in your set/list

Median

Range (subtract min\(X\) from max\(X\))

Variance: Take standard deviation \(\sigma_x\) and square it!

Mode: You still have to find that one on your own!

What's missing??

\[ \sigma : 27.309322 \]

\[ 745.7778 \]
TRY IT ALL TOGETHER NOW...

- Find the following for the set $X = \{2, 5, 8, 21, 45, 26, 5, 10\}$
  - Mean: $\bar{X} = 15.25$
  - Median: $\text{Median: } 9$
  - Mode: $5$
  - Range: $45 - 2 = 43$
  - Variance: $\sigma^2 = 187.4375$
  - Standard Deviation: $\sigma = 13.69078157$