7.4 Parallel Lines and Proportional Parts

Triangle Proportionality Theorem

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Example: If $BE \parallel CD$, then $\frac{AB}{BC} = \frac{AE}{ED}$.

In $\triangle PQR$, $ST \parallel RQ$. If $PT = 7.5$, $TQ = 3$, and $SR = 2.5$, find $PS$.

$$\frac{X}{2.5} = \frac{7.5}{3} \quad \text{and} \quad \frac{3X}{2.5} = \frac{18.75}{2.5} \quad \Rightarrow \quad x = 6.25$$

1. If $PS = 12.5$, $SR = 5$, and $PT = 15$, find $TQ$.

$$\frac{12.5X}{12.5} = \frac{75}{75} \quad \Rightarrow \quad x = 6$$

$$\frac{12.5}{5} = \frac{X}{X} \quad \Rightarrow \quad x = \frac{15}{5}$$
Converse of Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into proportional corresponding segments, then the line is parallel to the third side of the triangle.

Example: If \( \frac{AE}{EB} = \frac{CD}{DB} \), then \( \overline{AC} \parallel \overline{ED} \).

In \( \triangle DEF \), \( EH = 3 \), \( HF = 9 \), and \( DG \) is one-third the length of \( \overline{GF} \). Is \( \overline{DE} \parallel \overline{GH} \)?

\[
\frac{x}{3x} = \frac{3}{9} \\
\frac{1}{3} = \frac{1}{3} \\
3x = 9
\]

Yes

2. \( DG \) is half the length of \( \overline{GF} \), \( EH = 6 \), and \( HF = 10 \). Is \( \overline{DE} \parallel \overline{GH} \)?

\[
\frac{x}{2x} = \frac{6}{10} \\
\frac{1}{2} = \frac{3}{5}
\]

No
Proportional Parts of Parallel Lines

If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

**Example**  If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, then $\frac{AB}{BC} = \frac{EF}{FG}$.

Congruent Parts of Parallel Lines

If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

**Example**  If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, and $\overline{AB} \cong \overline{BC}$, then $\overline{EF} \cong \overline{FG}$.
4. **REAL ESTATE**  *Frontage* is the measurement of a property’s boundary that runs along the side of a particular feature such as a street, lake, ocean, or river. Find the ocean frontage for Lot A to the nearest tenth of a yard.

\[
\begin{align*}
6y - 5 &= \frac{4x + 3}{y} \\
2x - \frac{3}{3} + \frac{3}{3} &= \frac{3}{3} \\
\text{2x = 8} \\
\text{x = 4} \\
3y + 8 &= 5y - 7 \\
-3y + 7 - 3y + 7 &= 15 \\
\text{2y = 15} \\
\text{y = 15/2} \\
\end{align*}
\]
Triangle Midsegment Theorem

If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side, and is half its length.

\[ BD = \frac{1}{2}(AE) \]

\[ 2BD = AE \]
EXAMPLE Finding Lengths

In \( \triangle EFG \), \( H, J \), and \( K \) are midpoints.
Find \( HJ, JK \), and \( FG \).

\[ AB = 10 \text{ and } CD = 18. \text{ Find } EB, BC, \text{ and } AC. \]
EXAMPLE Identifying Parallel Segments

In \( \triangle DEF \), A, B, and C are midpoints. Name pairs of parallel segments.

\[ \overline{AC} \parallel \overline{EF} \quad \overline{BC} \parallel \overline{ED} \quad \overline{AB} \parallel \overline{DF} \]

Critical Thinking Find \( m \angle VUZ \). Justify your answer.

\[ \angle 65^\circ \parallel \overline{UZ} \]

Corresponding angles are

\[ \sim \]
20. **Indirect Measurement**  Kate wants to paddle her canoe across the lake. To determine how far she must paddle, she paced out a triangle, counting the number of strides, as shown.

a. If Kate’s strides average 3.5 ft, what is the length of the longest side of the triangle?

b. What distance must Kate paddle across the lake?
7.5 Parts of Similar Triangles

Triangle Angle Bisector

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

Example  If \( JM \) is an angle bisector of \( \triangle JKL \),
then \[ \frac{KM}{LM} = \frac{KJ}{LJ} \]
segments with vertex \( K \)
segments with vertex \( L \)

\[ \begin{align*}
11. \quad & \frac{x}{5} \times \frac{12}{10} \\
& 10x = 60 \\
& x = 6
\end{align*} \]

\[ \begin{align*}
12. \quad & \frac{x}{3} = \frac{8}{5} \\
& 5x = 24 \\
& x = \frac{24}{5}
\end{align*} \]

\[ \begin{align*}
13. \quad & \frac{14}{8} = \frac{x}{20} \\
& 280 = 8x \\
& x = 35
\end{align*} \]
Homework:
p.495 #1-7 and 10-13
p. 506 #20-23
Example 1
1. If XM = 4, XN = 6, and NZ = 9, find XY.
2. If XN = 6, XM = 2, and XY = 10, find NZ.

Example 2
3. In \( \triangle ABC \), BC = 15, BE = 6, DC = 12, and AD = 8. Determine whether \( DE \parallel AB \). Justify your answer.
4. In \( \triangle JKL \), JK = 15, JM = 5, LK = 13, and PK = 9. Determine whether \( JL \parallel MP \). Justify your answer.

\( \overline{GH} \) is a midsegment of \( \triangle KLM \). Find the value of \( x \).
5.
6.

7. MAPS Refer to the map at the right. 3rd Avenue and 5th Avenue are parallel. If the distance from 3rd Avenue to City Mall along State Street is 3201 feet, find the distance between 5th Avenue and City Mall along Union Street. Round to the nearest tenth.

10. If AB = 6, BC = 4, and AE = 9, find ED.
11. If AB = 12, AC = 16, and ED = 5, find AE.
12. If AC = 14, BC = 8, and AD = 21, find ED.
13. If AD = 27, AB = 8, and AE = 12, find BC.
ALGEBRA  Find $x$.

20.

21.

22.

23.
Quiz Topics (5 questions)

1. Ratios and Proportions

2. Equivalent fractions
\[ \frac{a}{b} = \frac{c}{d} \text{ then } \frac{b}{a} = \frac{d}{c} \]

3. Triangle Similarity
\[ \frac{a+b}{b} = \frac{c+d}{d} \]

4. Triangle Proportionality

5. Triangle Midsegment