Algebraic Proof  A list of algebraic steps to solve problems where each step is justified is called an algebraic proof. The table shows properties you have studied in algebra.

The following properties are true for any real numbers $a$, $b$, and $c$.

<table>
<thead>
<tr>
<th>Property</th>
<th>If $a = b$, then expression holds.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Property of Equality</td>
<td>$a + c = b + c$.</td>
</tr>
<tr>
<td>Subtraction Property of Equality</td>
<td>$a - c = b - c$.</td>
</tr>
<tr>
<td>Multiplication Property of Equality</td>
<td>$a \cdot c = b \cdot c$.</td>
</tr>
<tr>
<td>Division Property of Equality</td>
<td>$\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.</td>
</tr>
<tr>
<td>Reflexive Property of Equality</td>
<td>$a = a$.</td>
</tr>
<tr>
<td>Symmetric Property of Equality</td>
<td>If $a = b$, then $b = a$.</td>
</tr>
<tr>
<td>Transitive Property of Equality</td>
<td>If $a = b$ and $b = c$, then $a = c$.</td>
</tr>
<tr>
<td>Substitution Property of Equality</td>
<td>If $a = b$ and $b$ may be replaced by $a$ in any equation or expression.</td>
</tr>
<tr>
<td>Distributive Property</td>
<td>$a(b + c) = ab + ac$.</td>
</tr>
</tbody>
</table>

**Example** Solve $6x + 2(x - 1) = 30$. Write a justification for each step.

**Algebraic Steps**

1. $6x + 2(x - 1) = 30$
2. $6x + 2x - 2 = 30$
3. $8x - 2 = 30$
4. $8x - 2 + 2 = 30 + 2$
5. $8x = 32$
6. $\frac{8x}{8} = \frac{32}{8}$
7. $x = 4$

**Properties**

- Original equation or Given
- Distributive Property
- Substitution Property of Equality
- Addition Property of Equality
- Substitution Property of Equality
- Division Property of Equality

**Exercises**

Complete each proof.

1. **Given:** $\frac{4x + 6}{2} = 9$
   **Prove:** $x = 3$
   **Proof:**
   - a. $\frac{4x + 6}{2} = 9$
   - b. $2\left(\frac{4x + 6}{2}\right) = 2(9)$
   - c. $4x + 6 = 18$
   - d. $4x + 6 - 6 = 18 - 6$
   - e. $4x = 12$
   - f. $\frac{4x}{4} = \frac{12}{4}$
   - g. $x = 3$

   **Reasons**
   - a. Given
   - c. Subs.
   - e. Substitution
   - g. Substitution

2. **Given:** $4x + 8 = x + 2$
   **Prove:** $x = -2$
   **Proof:**
   - a. $4x + 8 = x + 2$
   - b. $4x + 8 - x = x + 2 - x$
   - c. $3x + 8 = 2$
   - d. $3x + 8 - 8 = 2 - 8$
   - e. $3x = -6$
   - f. $\frac{3x}{3} = \frac{-6}{3}$
   - g. $x = -2$

   **Reasons**
   - a. Given
   - c. Substitution
   - e. Substitution
   - g. Substitution
Geometric Proof  Geometry deals with numbers as measures, so geometric proofs use properties of numbers. Here are some of the algebraic properties used in proofs.

<table>
<thead>
<tr>
<th>Property</th>
<th>Segments</th>
<th>Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>AB = AB</td>
<td>m∠1 = m∠1</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If AB = CD, then CD = AB.</td>
<td>If m∠1 = m∠2, then m∠2 = m∠1.</td>
</tr>
<tr>
<td>Transitive</td>
<td>If AB = CD and CD = EF, then AB = EF.</td>
<td>If m∠1 = m∠2 and m∠2 = m∠3, then m∠1 = m∠3.</td>
</tr>
</tbody>
</table>

Example  Write a two-column proof to verify this conjecture.

Given: m∠1 = m∠2, m∠2 = m∠3
Prove: m∠1 = m∠3

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. m∠1 = m∠2</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. m∠2 = m∠3</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. m∠1 = m∠3</td>
<td>3. Transitive Property of Equality</td>
</tr>
</tbody>
</table>

Exercises

State the property that justifies each statement.

1. If m∠1 = m∠2, then m∠2 = m∠1.  Sym. Prop.
2. If m∠1 = 90 and m∠2 = m∠1, then m∠2 = 90.  Subs.
3. If AB = RS and RS = WY, then AB = WY.  Trans. Prop.
4. If AB = CD, then \( \frac{1}{2}AB = \frac{1}{2}CD \).  Mult. Prop.
5. If m∠1 + m∠2 = 110 and m∠2 = m∠3, then m∠1 + m∠3 = 110.  Subs.
7. If AB = RS and TU = WY, then AB + TU = RS + WY.  Add. Prop.
8. If m∠1 = m∠2 and m∠2 = m∠3, then m∠1 = m∠3.  Trans. Prop.
9. If the formula for the area of a triangle is \( A = \frac{1}{2}bh \), then bh is equal to 2 times the area of the triangle. Write a two-column proof to verify this conjecture.

Given: \( A = \frac{1}{2}bh \)
Prove: \( 2A = bh \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( A = \frac{1}{2}bh )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( 2A = 2\left(\frac{1}{2}bh\right) )</td>
<td>2. Mult. Property</td>
</tr>
</tbody>
</table>