9.5 Symmetry

**Line Symmetry**: figure can be mapped onto itself across a line (reflection symmetry or axis symmetry)

![Diagram of line symmetry](image)

State whether the figure has line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

1A. No

1B.

1C.

**Rotational Symmetry**: figure can be mapped onto itself by a rotation of 0° to 360° about the center of the figure (radial symmetry)

**Center of Symmetry**: center of the figure (point of symmetry)

**Order of Symmetry**: number of times it maps onto itself as it rotates from 0° to 360°

**Magnitude of Symmetry**: smallest angle that the figure can be rotated onto itself (angle of rotation)

Examples: The figure below has rotational symmetry because a rotation of 90°, 180°, or 270° maps the figure onto itself.

![Rotational symmetry examples](image)

Order = 4
Magnitude = 90°

\[
\frac{360°}{\text{order}} = \text{Magnitude}
\]
State whether the figure has rotational symmetry. Write yes or no. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

a. Yes; the regular hexagon has order 6 rotational symmetry and magnitude $360^\circ \div 6 = 60^\circ$. The center is the intersection of the diagonals.

b. No; no rotation between $0^\circ$ and $360^\circ$ maps the right triangle onto itself.

c. Yes; the figure has order 2 rotational symmetry and magnitude $360^\circ \div 2 = 180^\circ$. The center is the intersection of the diagonals.

**Key Concept: Three-Dimensional Symmetries**

**Plane Symmetry**
A three-dimensional figure has **plane symmetry** if the figure can be mapped onto itself by a reflection in a plane.

**Axis Symmetry**
A three-dimensional figure has **axis symmetry** if the figure can be mapped onto itself by a rotation between $0^\circ$ and $360^\circ$ in a line.

State whether the figure has plane symmetry, axis symmetry, both, or neither.

a. L-shaped prism

plane symmetry

b. regular pentagonal prism

both plane symmetry and axis symmetry
9.6 Dilations

**Dilation**: enlargement or reduction in respect to a center and scale factor

**Scale Factor** = \(\frac{\text{image}}{\text{pre-image}}\)

**Example 1** Draw a Dilation

Copy \(\triangle ABC\) and point \(D\). Then use a ruler to draw the image of \(\triangle ABC\) under a dilation with center \(D\) and scale factor \(\frac{1}{2}\).

**Step 1** Draw rays from \(D\) through each vertex.

**Step 2** Locate \(A'\) on \(\overrightarrow{DA}\) such that \(DA' = \frac{1}{2}DA\).

**Step 3** Locate \(B'\) on \(\overrightarrow{DB}\) and \(C'\) on \(\overrightarrow{DC}\) in the same way. Then draw \(\triangle A'B'C'\).

Copy the figure and point \(J\). Then use a ruler to draw the image of the figure under a dilation with center \(J\) and the scale factor \(k\) indicated.

1A. \(k = \frac{3}{2}\)

1B. \(k = 0.75\)
Determine whether the dilation from figure $W$ to $W'$ is an enlargement or a reduction. Then find the scale factor of the dilation and $x$.

15. **enlargement;**
   \[
   \frac{\text{image}}{\text{preimage}} = \frac{9}{4.5} = 2
   \]
   
   \[
   x = 4.5
   \]

16. **enlargement;**
   \[
   \frac{\text{image}}{\text{preimage}} = \frac{10}{5} = \frac{3}{6} = \frac{1}{2}
   \]
   
   \[
   x = 6
   \]

17. **reduction;**
   \[
   \frac{\text{image}}{\text{preimage}} = \frac{14}{10.5} = \frac{3}{4}
   \]

18. **reduction;**
   \[
   \frac{\text{image}}{\text{preimage}} = \frac{12}{4} = \frac{3}{1}
   \]

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**Homework**

p.655 #7, 9

p.666 #1-6, 8

p.677 #1-3
State whether the figure appears to have line symmetry. Write yes or no. If so, copy the figure, draw all lines of symmetry, and state their number.

1. 
2. 
3. no

State whether the figure has rotational symmetry. Write yes or no. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

4. no
5. 
6. yes; 4, 90°

1. yes; 4
6. yes; 4, 90°
3. yes; 1

7. **U.S. Capitol**  Completed in 1863, the dome is one of the most recent additions to the United States Capitol. It is supported by 36 iron ribs and has 108 windows, divided equally among three levels.

a. Excluding the spire of the dome, how many horizontal and vertical planes of symmetry does the dome appear to have? **no horizontal; 36 vertical**

b. Does the dome have axis symmetry? If so, state the order and magnitude of symmetry. **yes; 36; 10°**

8. State whether the figure has **plane** symmetry, **axis** symmetry, both, or **neither**. **both**
Copy the figure and point $M$. Then use a ruler to draw the image of the figure under a dilation with center $M$ and the scale factor $k$ indicated.

1. $k = \frac{1}{4}$

2. $k = 2$

3. Determine whether the dilation from Figure $B$ to $B'$ is an *enlargement* or a *reduction*. Then find the scale factor of the dilation and $x$.  

   *enlargement*; $\frac{4}{3}; 2$