Lesson 2.1: Union and Intersection of Sets

Pre-requisite Concepts: Whole Numbers, definition of sets, Venn diagrams

Objectives:
In this lesson, you are expected to:
1. Describe and define
   a. union of sets;
   b. intersection of sets.
2. Perform the set operations
   a. union of sets;
   b. intersection of sets.
3. Use Venn diagrams to represent the union and intersection of sets.

Note to the Teacher:
Below are the opening activities for students. Emphasize that just like with the whole number, operations are also used on sets. You may combine two sets or form subsets. Emphasize to students that in counting the elements of a union of two sets, elements that are common to both sets are counted only once.

Lesson Proper:
I. Activities
Answer the following questions:

1. Which of the following shows the union of set A and set B? How many elements are in the union of A and B?

   ![Diagram 1](image1)
   ![Diagram 2](image2)
   ![Diagram 3](image3)

2. Which of the following shows the intersection of set A and set B? How many elements are there in the intersection of A and B?

   ![Diagram 1](image1)
   ![Diagram 2](image2)
   ![Diagram 3](image3)
Here’s another activity:

Let

\[ V = \{ 2x \mid x \in I, 1 \leq x \leq 4 \} \]
\[ W = \{ x^2 \mid x \in I, -2 \leq x \leq 2 \} \]

What elements may be found in the intersection of V and W? How many are there? What elements may be found in the union of V and W? How many are there?

Do you remember how to use Venn Diagrams? Based on the diagram below, (1) determine the elements that belong to both A and B; (2) determine the elements that belong to A or B or both. How many are there in each set?
NOTE TO THE TEACHER:

Below are important terms, notations and symbols that students must remember. From here on, be consistent in your notations as well so as not to confuse your students. Give plenty of examples and non-examples.

**Important Terms/Symbols to Remember**

The following are terms that you must remember from this point on.

1. Let A and B be sets. The union of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

   An element $x$ belongs to the union of the sets A and B if and only if $x$ belongs to A or $x$ belongs to B. This tells us that $A \cup B = \{x \mid x \text{ is in } A \text{ or } x \text{ is in } B\}$

   Venn diagram:

   ![Venn Diagram](image)

Note to the Teacher:

Explain to the students that in general, the inclusive OR is used in mathematics. Thus, when we say, “elements belonging to A or B”, that includes the possibility that the elements belong to both. In some instances, “belonging to both” is explicitly stated when referring to the intersection of two sets. Advise students that from here onwards, OR is used inclusively.
2. Let A and B be sets. The intersection of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

An element $x$ belongs to the intersection of the sets A and B if and only if $x$ belongs to A and $x$ belongs to B. This tells us that $A \cap B = \{ x \mid x \text{ is in A and } x \text{ is in B} \}$

Venn diagram:

Sets whose intersection is an empty set are called disjoint sets.

3. The cardinality of the union of two sets is given by the following equation:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

II. Questions to Ponder (Post-Activity Discussion)

NOTE TO THE TEACHER

It is important for you to go over the answers of your students posed in the opening activities in order to process what they have learned for themselves. Encourage discussions and exchanges in the class. Do not leave questions unanswered. Below are the correct answers to the questions posed in the activities.

Let us answer the questions posed in the opening activity.

1. Which of the following shows the union of set A and set B? Set 2. This is because it contains all the elements that belong to A or B or both. There are 8 elements.

2. Which of the following shows the intersection of set A and set B? Set 3. This is because it contains all elements that are in both A and B. There are 3 elements.
In the second activity:
\[ V = \{ 2, 4, 6, 8 \} \]
\[ W = \{ 0, 1, 4 \} \]

Therefore, \( V \cap W = \{ 4 \} \) has 1 element and \( V \cup W = \{ 0, 1, 2, 4, 6, 8 \} \) has 6 elements. Note that the element \{ 4 \} is counted only once.

On the Venn Diagram: (1) The set that contains elements that belong to both A and B consists of two elements \{ 1, 12 \}; (2) The set that contains elements that belong to A or B or both consists of 6 elements \{ 1, 10, 12, 20, 25, 36 \}.

**NOTE TO THE TEACHER:**
Always ask for the cardinality of the sets if it is possible to obtain such number, if only to emphasize that
\[ n(A \cup B) \neq n(A) + n(B) \]
because of the possible intersection of the two sets. In the exercises below, use every opportunity to emphasize this. Discuss the answers and make sure students understand the “why” of each answer.

**III. Exercises**
1. Given sets A and B,

<table>
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<tr>
<th>Set A Students who play the guitar</th>
<th>Set B Students who play the piano</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethan Molina</td>
<td>Mayumi Torres</td>
</tr>
<tr>
<td>Chris Clemente</td>
<td>Janis Reyes</td>
</tr>
<tr>
<td>Angela Dominguez</td>
<td>Chris Clemente</td>
</tr>
<tr>
<td>Mayumi Torres</td>
<td>Ethan Molina</td>
</tr>
<tr>
<td>Joanna Cruz</td>
<td>Nathan Santos</td>
</tr>
</tbody>
</table>

determine which of the following shows (a) union of sets A and B; and (b) intersection of sets A and B?
Answers: (a) Set 4. There are 7 elements in this set. (b) Set 2. There are 3 elements in this set.

2. Do the following exercises. Write your answers on the spaces provided:

\[ A = \{0, 1, 2, 3, 4\} \quad B = \{0, 2, 4, 6, 8\} \quad C = \{1, 3, 5, 7, 9\} \]

Answers:

Given the sets above, determine the elements and cardinality of:

a. \( A \cup B = \{0, 1, 2, 3, 4, 6, 8\} \); \( n(A \cup B) = 7 \)
b. \( A \cup C = \{0, 1, 2, 3, 4, 5, 7, 9\} \); \( n(A \cup C) = 8 \)
c. \( A \cup B \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \); \( n(A \cup B \cup C) = 10 \)
d. \( A \cap B = \{0, 2, 4\} \); \( n(A \cap B) = 3 \)
e. \( B \cap C = \emptyset \); \( n(B \cap C) = 0 \)
f. \( A \cap B \cap C = \emptyset \); \( n(A \cap B \cap C) = 0 \)
g. \( (A \cap B) \cup C = \{0, 1, 2, 3, 4, 5, 7, 9\} \); \( n((A \cap B) \cup C) = 8 \)
NOTE TO THE TEACHER:
In Exercise 2, you may introduce the formula for finding the cardinality of the union of 3 sets. But, it is also instructive to give students the chance to discover this on their own. The formula for finding the cardinality of the union of 3 sets is:
\[ n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C). \]

3. Let \( W = \{ x | 0 < x < 3 \}, \ Y = \{ x | x > 2 \}, \) and \( Z = \{x | 0 \leq x \leq 4 \}. \) Determine (a) \((W \cup Y) \cap Z;\) (b) \(W \cap Y \cap Z.\)

Answers:
Since at this point students are more familiar with whole numbers and fractions greater than or equal to 0, use a partial real numberline to show the elements of these sets.

(a) \((W \cup Y) \cap Z = \{x | 0 < x \leq 4\}\)
(b) \(W \cap Y \cap Z = \{x | 2 < x < 3\}\)

NOTE TO THE TEACHER:
End with a good summary. Provide more exercises on finding the union and intersection of sets of numbers.

Summary
In this lesson, you learned about the definition of union and intersection of sets. You learned also how to use Venn diagrams to represent the unions and the intersection of sets.