AP Calculus Free-Response Questions

1. Consider the following functions defined for all x: $f_1(x) = x$, $f_2(x) = x\cos x$, $f_3(x) = 3e^{2x}$, $f_4(x) = x - |x|$. Answer the following questions (a, b, c, and d) about each of these functions. Indicate your answer by writing either yes or no in the appropriate space in the given rectangular grid. No justification is required but each blank space will be scored as an incorrect answer.

Questions	f_1	f_2	f_3	f_4
a. Does f(-x) = -f(x)				
b. Does the inverse function exist for all x?				
c. Is the function periodic?				
d. Is the function continuous at x = 0?				

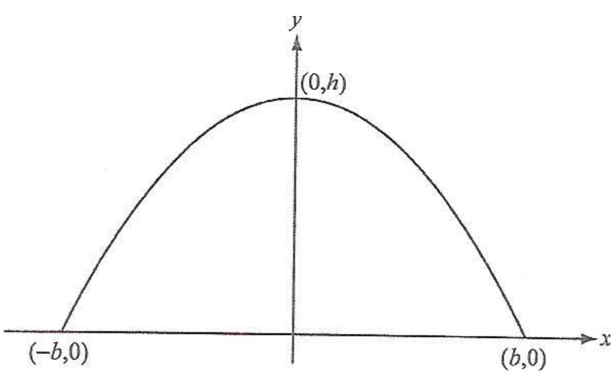
2. A particle moves along the x-axis in such a way that its position at time t is given by $x(t) = 3t^4 - 16t^3 + 24t^2$, for $-5 \leq t \leq 5$.
   a. Determine the velocity and acceleration of the particle at time t.
   b. At what values of t is the particle at rest?
   c. At what values of t does the particle change direction?
   d. What is the velocity when the acceleration is first zero?

3. Given f(x) = $\frac{1}{x}$ + ln x, defined only on the closed interval $\frac{1}{e} \leqslant x \le e$,

   a. Showing your reasoning, determine the value of x at which f has its
      (i) absolute maximum,
      (ii) absolute minimum.
   b. For what values of x is the curve concave up?
   c. On the coordinate axes provided, sketch the graph of f over the interval $\frac{1}{e} \leqslant x \le e$.
   d. Given that the mean value (average ordinate) of f over the interval is $\frac{2}{e-1}$, state in words a geometrical interpretation of this number relative to the graph.

4. The number of bacteria in a culture at time t is given approximately by y = $1000(25 + te^{-t/20})$ for $0 \leqslant t \le 100$.
   a. Find the largest number and the smallest number of bacteria in the culture during the interval.
   b. At what time in the interval is the rate of change in the number of bacteria a minimum?

5. Let R denote the region enclosed between the graph of $y = x^2$ and the graph of y = 2x.
   a. Find the area of region R.
   b. Find the volume of the solid obtained by revolving the region R about the y-axis.



6. An arched window with base width 2b and height h is set into a wall. The arch is to be either an arc of a parabola or a half-cycle of a cosine curv.
   a. If the arch is an arc of a parabola, write an equation for the parabola relative to the coordinate system shown in the figure.

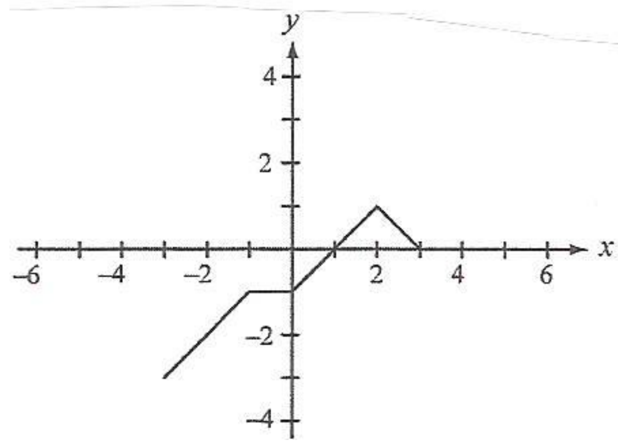
7. a. Sketch the graph of y = $\frac{e^x + e^{-x}}{2}$.
   b. Let R be a point on the curve and let the x-coordinate of R be r ($r \neq 0$). The tangent line to the curve at R crosses the x-axis at a point Q. Find the coordinates of Q.
   c. If P is the point (r, 0), find the length of PQ as function of r and the limiting value of this length as r increases without bound.

8. Given the parabola $y = x^2 - 2x + 3$:

a. find an equation for the line L, which contains the point (2,3) and is perpendicular to the line tangent to the parabola at (2,3).

b. find area of that part of the first quadrant which lies below <u>both</u> the line L and the parabola.

9. A function f is defined on the closed interval from -3 to 3 and has the graph shown below.



a. Sketch the entire graph of $y = |f(x)|$.

b. Sketch the entire graph of $y = f(|x|)$.

c. Sketch the entire graph of $y = f(-x)$.

d. Sketch the entire graph of $y = f(\frac{1}{2}x)$

e. Sketch the entire graph of $y = f(x - 1)$

10. Consider the function f given by $f(x) = x^{4/3} + 4x^{1/3}$ on the interval $-8 \leq x \leq 8$.

a. Find the coordinates of all points at which the tangent to the curve is a horizontal line.

b. Find the coordinates of all points at which the tangent to the curve is a vertical line.

c. Find the coordinates of all points at which the absolute maximum and absolute minimum occur.

d. For what values of x is this function concave down?

e. Sketch the graph of the function on this interval.

11. A right circular cone and a hemisphere have the same base, and the cone is inscribed in the hemisphere. The figure is expanding in such a way that the combined surface area of the hemisphere and its base is increasing at a constant rate of 18 square inches per second. At what rate is the volume of the cone changing at the instant when the radius of the common base is 4 inches. Show your work.

<u>NOTE</u>: The surface area of a sphere of radius r is $S = 4\pi r^2$ and the volume of a right circular cone oh height h and base radius r is $V = \frac{1}{3}\pi r^2 h$.

12. A particle moves along the x-axis in such a way that at time t > 0 its position coordinate is $x = \sin(e^t)$.
   a. Find the velocity and acceleration of the particle at time t.
   b. At what time does the particle first have zero velocity?
   c. What is the acceleration of the particle at the time determined in part b?

13. A parabola P is symmetric to the y-axis and passes through (0, 0) and (b, e^{-b^2}) where b > 0.
   a. Write an equation for P.
   b. The closed region bounded by P and the line y = e^{-b^2} is revolved about the y-axis to form a solid figure F. Compute the volume of F.
   c. For what value of b is the volume of F a maximum? Justify your answer.

14. From the fact that sint $\leqslant$ t for all t $\geq$ 0, use integration repeatedly to prove the following inequalities. Show your work.

$$1 - \frac{1}{2!}x^2 \leq \cos x \leq 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 \text{ for all } x \geq 0.$$

15. Let f(x) = ln(x) for all x >0, and let g(x) = x^2 - 4 for all real x. Let H be the composition of f with g, that is, H(x) =f(g(x)). Let K be the composition of g with f, that is, K(x)= g(f(x)).
   a. Find the domain of H.
   b. Find the range of H.
   c. Find the domain of K.
   d. Find the range of K.
   e. Find H′(7).

16. Let R be the region in the first quadrant that lies below both of the curves y =$3x^2$ and y = $\frac{3}{x}$ and to the left of the line x = k, where k > 1.
   a. Find the area of R as a function of k.
   b. When the area of R is 7, what is the value of k?
   c. If the area of R is increasing at the constant rate of 5 square units per second at what rate is k increasing when k = 15?

17. Consider F(x)= $\cos^2 x$ + 2cos x over one complete period beginning with x = 0.
   a. Find all values of x in this period at which F(x) = 0.
   b. Find all values of x in this period at which the function has a minimum. Justify your answer.
   c. Over what intervals in this period is the curve concave up?

18. Find the area of the largest rectangle (with sides parallel to the coordinate axes) that can be inscribed in the region enclosed by the graphs of f(x) = 18 - x^2 and G(x) = $2x^2$ - 9.

19. Let R be the region of the first quadrant bounded by the x-axis and the curve y = 2x - x^2.

a. Find the volume produced when R is revolved around the x-axis.
b. Find the volume produced when R is revolved around the y-axis.

20. A particle starts at the point (5,0) at t=0 and moves along the x-axis in such a way that at time t > 0 its velocity v(t) is given by v(t) = $\frac{t}{1+t^2}$.

a. Determine the maximum velocity attained by the particle. Justify your answer.
b. Determine the position of the particle at t = 6.
c. Find the limiting value of the velocity as t increases without bound.
d. Does the particle ever pass the point (500,0)? Explain.

21. Let f be the function defined by f(x) = |x| $.5e^{-x^2}$ for all real numbers x.

a. Describe the symmetry of the graph of f.
b. Over what intervals of the domain is this function increasing?
c. Sketch the graph of f on the axes provided showing clearly:
   (i) behavior near the origin
   (ii) maximum and minimum points
   (iii) behavior for large |x|.

22. Let f(x) = $4x^3$ - 3x - 1.

a. Find the x-intercepts of the graph of f.
b. Write an equation for the tangent line to the graph of f at x = 2.
c. Write an equation of the graph that is the reflection across the y-axis of the graph of f.

23. A particle starts at time t = 0 and moves on a number line so that its position at time t is given by x(t) = $(t - 2)^3$ (t -6).

a. When is the particle moving to the right?
b. When is the particle at rest?
c. When does the particle change direction?
d. What is the farthest to the left of the origin that the particle moves?

24. Let f(x) = k sin (kx), where k is a positive constant.

a. Find the area of the region bounded by one arch of the graph of f and the x-axis.
b. Find the area of the triangle formed by the x-axis and the tangent to one arch of f at the points where the graph of f crosses the x-axis.

25. A man has 340 yards of fencing for enclosing two separate fields, one of which is to be a rectangle twice as long as it is wide and the other a square. The square field must contain at least 100 square

yards and the rectangular one must contain at least 800 square yards.

a. If x is the width of the rectangular field, what are the maximum and minimum possible values of x?

b. What is the greatest number of square yards that can be enclosed in the two fields? Justify Your answer.

26. Let y = 2 $e^{\cos(x)}$.

a. Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b. If x and y both vary with time in such a way that y increases at a steady rate of 5 units per second, at what rate is x changing when x = $\frac{\pi}{2}$.

27. The shaded region R is bounded by the graphs of xy =1, x = 1, x = 2, and y = 0.

a. Find the volume of the solid figure generated by revolving the region R about the x-axis.

b. Find the volume of the solid figure generated by revolving the region R about the line x = 3.

28. A function f is defined for all real numbers and has the following three properties:

(i) f(1) = 5,

(ii) f(3) = 21, and

(iii) for all real values of a and b, f(a + b) - f(a) = kab + 2b^2 where k is a fixed real number independent of a and b.

a. Use a = 1 and b = 2 to find the value of k.

b. Find f′(3).

c. Find f′ (x) and f(x) for all real x.

29. Given f(x) = x^3 -6x^2 + 9x and g(x) = 4.

a. Find the coordinates of the points common to the graphs of f and g.

b. Find all the zeros of f.

c. If the domain of f is limited to the closed interval [0,2], what is the range of f? Show your reasoning.

30. A particle moves on the x-axis so that its acceleration at any time t > 0 is given by a = $\frac{t}{8} - \frac{1}{t^2}$.

When t = 1, v = $\frac{9}{16}$, and s = $\frac{25}{48}$.

a. Find the velocity v in terms of t.

b. Does the numerical value of the velocity ever exceed 500? Explain.

c. Find the distance s from the origin at time t = 2.

31. Given the curve x + xy + 2y^2 = 6.

a. Find an expression for the slope of the curve at any point (x,y) on the curve.

b. Write an equation for the line tangent to the curve at the point (2,1).

c. Find the coordinate of all other points on this curve with slope equal to the slope at (2,1).

32. a. What is the set of all values of b for which the graphs of y = 2x + b and y^2 = 4x intersect in two distinct points?

b. In the case b = -4, find the area of the region enclosed by $y = 2x - 4$ and $y^2 = 4x$.

c. In the case b = 0, find the volume of the solid generated by revolving about the x-axis the region bounded by $y = 2x$ and $y^2 = 4x$.

33. a. Find the coordinate of the absolute maximum point for the curve $y = x e^{-kx}$ where k is a fixed positive number. Justify your answer.

b. Write an equation for the set of absolute maximum points for the curves $y = x e^{-kx}$ as k varies through positive values.

34. A manufacturer finds it costs him $x^2 + 5x + 7$ dollars to produce x tons of an item. At production levels above 3 tons, he must hire additional workers, and his costs increase by $3(x - 3)$ dollars on his total production. If the price he receives is \$13 per ton regardless of how much he manufactures and if he has a plant capacity of 10 tons, what level of output maximizes his profits?

35. a. Find the area A, as a function of k, of the region in the first quadrant enclosed by the y-axis and the graphs of $y = \tan x$ and $y = k$ for $k > 0$.

b. What is the value of A when k = 1?

c. If the line y = k is moving upward at the rate of $\frac{1}{10}$ units per second, at what rate is A changing when k = 1?

36. Given $f(x) = |\sin x|$, $-\pi \le x \le \pi$, and $g(x) = x^2$ for all real x.

a. On the axes provided, sketch the graph of f.

b. Let $H(x) = g(f(x))$. Write an expression for H(x).

c. Find the domain and range of H.

d. Find an equation of the line tangent to the graph of H at the point where $x = \pi/4$.

37. Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$. The graph of $y = P(x)$ is symmetric with respect to the y-axis, has a relative maximum at (0,1), and has an absolute minimum at (q, -3).

a. Determine the values of a, b, c, and d, and using these values write an expression for P(x).

b. Find all possible values for q.

38. Let $f(x) = kx^2 + c$.

a. Find x_o in terms of k such that the tangent lines to the graph of f at $(x_o, f(x_o))$ and $(-x_o, f(-x_o))$ are perpendicular.

b. Find the slopes of the tangent lines mentioned in a.

c. Find the coordinates, in terms of k and c, of the point of intersection of the tangent lines mentioned in a.

39. Let f be a function defined for all x > -5 and having the following properties.

(i) $f''(x) = \frac{1}{3\sqrt{x+5}}$ for all x in the domain of f.

(ii) The line tangent to the graph of f at (4,2) has an angle of inclination of 45°.
Find an expression for f(x).

40. A ball is thrown from the origin of a coordinate system. The equation of its path is $y = mx - (1/1000)e^{2m}x^{2}$, where m is positive and represents the slope of the path of the ball at the orig

a. For what value of m will the ball strike the horizontal axis at the greatest distance from the origin? Justify your answer.

b. For what value of m will the ball strike at the greatest height on a vertical wall located 100 feet from the origin?

41. Given two functions f and g defined by f(x) = tan (x) and g(x) = $\sqrt{2}$ cos x.

a. Find the coordinates of the point of intersection of the graphs of f and g in the interval $0 < x < (\pi/2)$.

b. Find the area of the region enclosed by the y-axis and the graphs of f and g.

42. The rate of change in the number of bacteria in a culture is proportional to the number present. In a certain laboratory experiment, a culture had 10,000 bacteria initially, 20,000 bacteria at time t_1 minutes, and 100,000 bacteria at $(t_1 + 10)$ minutes.

a. In terms of t only, find the number of bacteria in the culture at any time t minutes, $t \geq 0$.

b. How many bacteria were there after 20 minutes?

c. How many minutes had elapsed when the 20,000 bacteria were observed?

43. Given the function f defined by $f(x) = \ln(x^2 - 9)$.

a. Describe the symmetry of the graph of f.

b. Find the domain of f.

c. Find all values of x such that f(x) = 0.

d. Write a formula for $f^{-1}(x)$, the inverse function of f, for x > 3.

44. A particle moves along the x-axis in such a way that its position at time t for $t \geq 0$ is given by $x = \frac{1}{3}t^3 - 3t^2 + 8t$.

a. Show that at time t= 0 the particle is moving to the right.

b. Find all values of t for which the particle is moving to the left.

c. What is the position of the particle at time t = 3?

d. When t = 3, what is the total distance the particle has traveled?

45. Given the function f defined for all real numbers by $f(x) = 2|x - 1|\,x^2$.

a. What is the range of the function?
b. For what values of x is the function continuous?
c. For what values of x is the derivative of f(x) continuous?
d. Determine the $\int_0^1 f(x)dx$.

46. Given the function defined by y = x + sin(x) for all x such that $\pi/2 \leq x \leq 3\pi/2$.
a. Find the coordinates of all maximum and minimum points on the given interval. Justify your answers.
b. Find the coordinates of all points of inflection on the given interval. Justify your answers.
c. On the axes provided, sketch the graph of the function.

47. The line x = c where c > 0 intersects the cubic $y = 2x^3 + 3x^2 - 9$ at point P and the parabola $y = 4x^2 + 4x + 5$ at point Q.
a. If a line tangent to the cubic at point P is parallel to the line tangent to the parabola at point Q, find the value of c where c > 0.
b. Write the equations of the two tangent lines described in a.

48. Let R be the region in the first quadrant bounded by the graphs of $\frac{x^2}{9}+\frac{y^2}{81}=1$ and 3x + y = 9.
a. Set up but do not integrate an integral representing the area of R. Express the integrand as a function of a single variable.
b. Set up but do not evaluate an integral representing the volume of the solid generated when R is rotated about the x-axis. Express the integrand as a function of a single variable.
c. Set up but do not evaluate an integral representing the volume of the solid generated when R is rotated about the y-axis. Express the integrand as a function of a single variable.

49. Given a function f with the following properties:
(i) $f(x + h) = e^h f(x) + e^x f(h)$ for all real numbers x and h.
(ii) f(x) has a derivative for all real numbers x.
(iii) f′(0) = 2.
a. Show that f(0) = 0.
b. Using the definition of f′(0), find $\lim \frac{f(x)}{x}$ as $x \to 0$.
c. Prove there exists a real number p such that $f'(x) = f(x) + p e^x$ for all real numbers x.
d. What is the value of the number p that is described in c?

50. Let f be a real-valued function defined by f(x) = $\sqrt{1+6x}$.

a. Give the domain and range of f.
b. Determine the slope of the line tangent to the graph of f at x = 4.
c. Determine the y-intercept of the line tangent to the graph of f at x = 4.
d. Give the coordinate of the point on the graph of f where the tangent line is parallel to y = x + 12.

51. Given the two functions f and h such that $f(x) = x^3 - 3x^2 - 4x + 12$ and

h(x) = { $h(x) = \frac{f(x)}{x-3}$ for $x \neq 3$, and p for x = 3}

a. Find all zeros of the function f.
b. Find the value of p so that the function h is continuous at x = 3. Justify your answer.
c. Using the value of p found in b, determine whether h is an even function. Justify your answer.

52. Let R be the region bounded by the curves $f(x) = \frac{4}{x}$ and $g(x) = (x - 3)^2$.

a. Find the area of R.
b. Find the volume of the solid generated by revolving R about the x-axis.

53. a. A point moves on the hyperbola $3x^2 - y^2 = 23$ so that its y-coordinate is increasing at a constant rate of 4 units per second. How fast is the x-coordinate changing when x = 4?

b. For what values of k will the line 2x + 9y + k= 0 be normal to the hyperbola $3x^2 - y^2 = 23$?

54. Given the function defined by $y = e^{\sin(x)}$ for all x such that $-\pi \le x \le 2\pi$.

a. Find the x- and y-coordinates of all maximum and minimum points on the given interval. Justify your answers.
b. On the axes provided, sketch the graph of the function.
c. Write an equation for the axis of symmetry of the graph

55. a. Given $5x^3 + 40 = \int_x^c f(t)\, dt$.

(i) Find f(x).
(ii) Find the value of c.

b. If $F(x) = \int_x^3 \sqrt{1+t^{16}}\, dt$, find F′ (x).

56. For a differentiable function f, let f*(x) = limit $\frac{f(x+h) - f(x-h)}{h}$ as $h \to 0$.

a. Determine f*(x) for $f(x) = x^2 + x$.
b. Determine f*(x) for f(x) = cos(x).
c. Write an equation that expresses the relationship between the functions f* and f′, where f′ denotes the usual derivative of f. (The BC exam requires justification of the answer)

57. Let f(x) = cos(x) for $0 \le x \le 2\pi$, and let g(x) = ln(x) for all x > 0. Let S be the composition of g with f,

that is, S(x) = g(f(x)).

a. Find the domain of S.

b. Find the range of S.

c. Find the zeros of S.

d. Find the slope of the line tangent to the graph of S at x = $\frac{\pi}{3}$.

58. Consider the function f defined by $f(x) = (x^2 - 1)^3$ for all real numbers x.

a. For what values of x is the function increasing?

b. Find the x- and y-coordinates of the relative maximum and minimum points. Justify your answer.

c. For what values of x is the graph of f concave upward?

d. Using the information found in parts a, b, and, c, sketch the graph of f on the axes provided.

59. Given the function f defined for all real numbers by f(x) = $e^{\frac{x}{2}}$.

a. Find the area of the region R bounded by the line y = e, the graph of f, and the y-axis.

b. Find the volume of the solid generated by revolving R, the region in a, about the x-axis.

60. Let f and g and their inverses f^{-1} and g^{-1} be differentiable functions and let the values of f, g, and the derivatives f′ and g′ at x = 1 and x = 2 be given by the table below:

x	1	2
f(x)	2	3
g(x)	2	π
f′ (x)	5	6
g′ (x)	4	7

Determine the value of each of the following:

a. The derivative of f + g at x = 2.

b. The derivative of fg at x = 2.

c. The derivative of f/g at x = 2.

d. h′(1) where h(x) = f(g(x)).

e. The derivative of g^{-1} at x = 2.

61. A particle moves along the x-axis with acceleration given by $a(t) = 2t - 10 + \frac{12}{t}$ for t $\geq$ 1.

a. Write an expression for the velocity v(t), given that v(1) = 9.

b. For what values of t, 1 $\leq$ t $\leq$ 3, is the velocity a maximum?

c. Write an expression for the position x(t), given that x(1) = -16.

62. A rectangle has a constant area of 200 square meters and its length L is increasing at the rate of 4 meters per second.

a. Find the width W at the instant the width is decreasing at the rate of 0.5 meters per second.

b. At what rate is the diagonal D of the rectangle changing at the instant when the width is 10 meters.

63. Let f be the real-valued function defined by $f(x) = \sin^3(x) + \sin^3|x|$.

a. Find $f'(x)$ for $x > 0$.

b. Find $f'(x)$ for $x < 0$.

c. Determine whether f(x) is continuous at x = 0. Justify your answer.

d. Determine whether the derivative of f(x) exists at x = 0. Justify your answer.

64. Given the function f defined by $f(x) = x^3 - x^2 - 4x + 4$.

a. Find the zeros of f.

b. Write an equation of the line tangent to the graph of f at x = -1.

c. The point (a,b) is on the graph of f and the line tangent to the graph at (a,b) passes through the point (0,-8) which is not on the graph of f. Find the value of a and b.

65. Let $f(x) = (1 - x)^2$ for all real numbers x, and let $g(x) = \ln(x)$ for all $x > 0$. Let $h(x) = (1 - \ln(x))^2$.

a. Determine whether h(x) is the composition f(g(x)) or the composition g(f(x)).

b. Find $h'(x)$.

c. Find $h''(x)$.

d. On the axes provided, sketch the graph of h.

66. Given the function f defined by $f(x) = \dfrac{2x-2}{x^2+x-2}$.

a. For what values of x is f(x) discontinuous?

b. At each point of discontinuity found in part a, determine whether f(x) has a limit and, if so, give the value of the limit.

c. Write an equation for each vertical and horizontal asymptote to the graph of f. Justify your answer.

d. A rational function $g(x) = \dfrac{a}{b+x}$ is such that g(x) = f(x) wherever f is defined. Find the value of a and b.

67. A particle moves on the x-axis so that its velocity at any time t is given by $v(t) = \sin(2t)$. At t = 0, the particle is at the origin.

a. For $0 \le t \le \pi$, find all values of t for which the particle is moving to the left.

b. Write an expression for the position of the particle at any time t.

c. For $0 \le t \le \dfrac{\pi}{2}$, find the average value of the position function determined in part b.

68. Given the curve $x^2 - xy + y^2 = 9$.

a. Write a general expression for the slope of the curve.

b. Find the coordinates of the points on the curve where the tangents are vertical.

c. At the point (0, 3) find the rate of change in the slope of the curve with respect to x.

69. Given the function f defined by f(x) = e^{-x^2} .

a. Find the maximum area of the rectangle that has two vertices on the x-axis and two on the graph of f. Justify your answer.

b. Let R be the region in the first quadrant bounded by the x- and y-axes, the graph of f, and the line x = k. Find the volume of the solid generated by revolving R about the y-axis.

c. Evaluate the limit of the volume determined in part b as k increases without bound.

70. Let g and h be any two twice-differentiable functions that are defined for all real numbers and that satisfy the following properties for all x:

(i) $(g(x))^2 + (h(x))^2 = 1$

(ii) $g'(x) = (h(x))^2$

(iii) $h(x) > 0$

(iv) $g(0) = 0$

a. Justify that $h'(x) = -g(x)h(x)$ for all x.

b. Justify that h has a relative maximum at x = 0.

c. Justify that the graph of g has a point of inflection at x = 0.

71. Given the function f defined by $f(x) = 2x^3 - 3x^2 - 12x + 20$.

a. Find the zeros of f.

b. Write an equation of the line normal to the graph of f at x = 0.

c. Find the x- and y-coordinates of all points on the graph of f where the line tangent to the graph is parallel to the x-axis.

72. A function f is defined by $f(x) = x e^{-2x}$ with domain $0 \le x \le 10$.

a. Find all values of x for which the graph of f is increasing and all values of x for which the graph decreasing.

b. Give the x- and y-coordinates of all absolute maximum and minimum points on the graph of f. Justify your answers.

73. Find the maximum volume of a box that can be made by cutting out squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides. Justify your answer.

74. A particle moves along a line so that at any time t its position is given by $x(t) = 2\pi t + \cos 2\pi t$.

a. Find the velocity at time t.

b. Find the acceleration at time t.

c. What are all values of t, $0 \leq t \leq 3$, for which the particle is at rest?

d. What is the maximum velocity?

75. Let R be the region bounded by the graph of $y = (1/x) \ln(x)$, the x-axis, and the line x = e.

a. Find the area of the region R.

b. Find the volume of the solid formed by revolving the region R about the y-axis.

76. Given the function f where $f(x) = x^2 - 2x$ for all real numbers x.

a. On the axes provided, sketch the graph of $y = |f(x)|$.

b. Determine whether the derivative of $|f(x)|$ exists at x = 0. Justify your answer.

c. On the axes provided, sketch the graph of $y = f(|x|)$.

d. Determine whether $f(|x|)$ is continuous at x = 0. Justify your answer.

77. Let f be the function defined by $f(x)\ x^3 + ax^2 + bx + c$ and having the following properties:

(i) The graph of f has a point of inflection at (0,-2).

(ii) The average (mean) value of f(x) on the closed interval [0,2] is -3.

a Determine the values of a, b, and c.

b. Determine the value of x that satisfies the conclusion of the Mean Value Theorem for f on the closed interval [0,3].

78. Let R be the region enclosed by the graphs of $y = x^3$ and $y\ \sqrt{x}$.

a. Find the area of R.

b. Find the volume of the solid generated by revolving R about the x-axis.

79. A rectangle ABCD with sides parallel to the coordinate axes is inscribed in the region enclosed by the graph of $y = -4x^2 + 4$ and the x-axis.

a. Find the x- and y-coordinates of C so that the area of the rectangle ABCD is a maximum.

b. The point C moves along the curve with its x-coordinate increasing at the constant rate of 2 units per second. Find the rate of change of the area of rectangle ABCD when $x = \frac{1}{2}$.

80. Let $f(x) = \ln(x^2)$ for $x > 0$ and $g(x) = e^{2x}$ for $x \geq 0$. Let H be the composition of f with g, that is H(x) = f(g(x)), and let K be the composition of g with f, that is, K(x) = g(f(x)).

a. Find the domain of H and write an expression for H(x) that does not contain the exponential function.

b. Find the domain of K and write an expression for K(x) that does not contain the exponential function.

c. Find an expression for $f^{-1}(x)$, where f^{-1} denotes the inverse function of f, and find the domain of f^{-1}.

81. The acceleration of a particle moving along a straight line is given by $a = 10e^{2t}$.

a. Write an expression for the velocity v, in terms of time t, if v = 5 when t = 0.
b. During the time when the velocity increases from 5 to 15, how far does the particle travel?
c. Write an expression for the position s, in terms of time t, of the particle if s = 0 when t = 0.

82. Given the function f defined by $f(x) = \cos(x) - \cos^2 x$ for $-\pi \leq x \leq \pi$.

a. Find the x-intercepts of the graph of f.
b. Find the x- and y-coordinates of all relative maximum points of f. Justify your answer.
c. Find the intervals on which the graph of f is increasing.
d. Using the information found in parts a, b, and c, sketch the graph of f on the axes provided.

83. Let y = f(x) be the continuous function that satisfies the equation $x^4 - 5x^2y^2 + 4y^4 = 0$ and whose graph contains the points (2,1) and (-2,-2). Let $\underline{\ell}$ be the line tangent to the graph of f at x = 2.

a. Find an expression for y′.
b. Write an equation for line $\underline{\ell}$.
c. Give the coordinates of a point that is on the graph of f but is not on line $\underline{\ell}$.
d. Give the coordinates of a point that is on line $\underline{\ell}$ but is not on the graph of f.

84. Let p and q be real numbers and let f be the function defined by:

$$f(x) = \begin{cases} 1 + 2p(x-1) + (x-1)^2, & for\ x \leq 1 \\ qx + p, & for\ x > 1 \end{cases}$$

a. Find the value of q in terms of p, for which f is continuous at x = 1.
b. Find the values of p and q for which f is differentiable at x = 1.
c. If p and q have the values determined in part (b), is f″ a continuous function? Justify your answer.

85. Let f be the function defined by $f(x) = x^4 - 3x^2 + 2$.

a. Find the zeros of f.
b. Write an equation of the line tangent to the graph of f at the point where x = 1.
c. Find the x-coordinate of each point at which the line tangent to the graph of f is parallel to the line y = -2x + 4.

86. Let R be the region in the <u>first quadrant</u> enclosed by the graphs of $y = 4 - x^2$, y = 3x, and the <u>y-axis</u>.

a. Find the area of the region R.
b. Find the volume of the solid formed by revolving the region R about the <u>x-axis.</u>

87. Let f be the function defined by $f(x) = 12x^{(2/3)} - 4x$.

a. Find the intervals on which f is increasing.
b. Find the x- and y-coordinates of all relative maximum points.
c. Find the x- and y-coordinates of all relative minimum points.
d. Find the intervals on which f is concave downward.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

88. Let f be the function defined by $f(x) = 5^{\sqrt{2x^2-1}}$.

a. Is f an even or odd function? Justify your answer.
b. Find the domain of f.
c. Find the range of f.
d. Find $f'(x)$.

89. Let f be a function defined by

$$f(x) = \begin{cases} 2x + 1, & \text{for } x \le 2, \\ \frac{1}{2}x^2 + k, & \text{for } x > 2. \end{cases}$$

a. For what values of k will f be continuous at $x = 2$? Justify your answer.
b. Using the value of k found in part a, determine whether f is differentiable at $x = 2$. Use the definition of the derivative to justify your answer.
c. Let $k = 4$. Determine whether f is differentiable at $x = 4$. Justify your answer.

90. A particle moves along the x-axis so that at time t its position is given by
$x(t) = \sin(\pi t^2)$ for $-1 \le t \le 1$.

a. Find the velocity at time t.
b. Find the acceleration at time t.
c. For what values of t does the particle change direction?
d. Find all values of t for which the particle is moving to the left.

91. Let f be a continuous function that is defined for all real numbers x and that has the following properties.

(i) $\int_1^3 f(x)\,dx = \frac{5}{2}$

(ii) $\int_1^5 f(x)\,dx = 10$

a. Find the average (mean) value of f over the closed interval [1,3].
b. Find the value of $\int_3^5 (2f(x)+6)\,dx$.
c. Given that $f(x) = ax + b$, find the values of a and b.

92. A particle moves along the x-axis in such a way that its acceleration at time t for t > 0 is given by $a(t) = \frac{3}{t^2}$. When t = 1, the position of the particle is 6 and the velocity is 2.

a. Write an equation for the velocity, v(t), of the particle for all t > 0.
b. Write an equation for the position, x(t), of the particle for all t > 0.
c. Find the position of the particle when t = e.

93. Given that f is the function defined by $f(x) = \frac{x^3 - x}{x^3 - 4x}$.

a. Find the limit as x approaches 0 of f(x).
b. Find the zeros of f.
c. Write an equation for each vertical and each horizontal asymptote to the graph of f.
d. Describe the symmetry of the graph of f.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

94. Let R be the region in the first quadrant that is enclosed by the graph of y = tan(x), the x-axis, and the line $x = \frac{\pi}{3}$.

a. Find the area of R.
b. Find the volume of the solid formed by revolving R about the x-axis.

95. A ladder 15 feet long is leaning against a building so that the end X is on level ground and end Y is on the wall. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

a. Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
b. Find the rate of change in square feet per second of the area of the triangle XOY when X is 9 feet from the building.

96. Let f be the function defined by $f(x) = (x^2 + 1)e^{-x}$ for $-4 \le x \le 4$.

a. For what value of x does f reach its absolute maximum? Justify your answer.
b. Find the x-coordinates of all points of inflection of f. Justify your answer.

97. A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and volume is 36 cubic meters. If building the tank costs $10 per square meter for the base and $5 per square meter for the sides, what is the cost of the least expensive tank?

98. For all real numbers x, f is a differentiable function such that f(-x) = f(x). Let f(p) = 1 and f′(p) = 5 for some p > 0.

a. Find f′(p).
b. Find f′(0).
c. If l_1 and l_2 are lines tangent to the graph of f at (-p,1) and (p,1), respectively, and if l_1 and l_2 intersect at point Q, find the x- and y-coordinates of Q in terms of p

Scientific calculators permitted

99. Let f be the function defined by $f(x) = -2 + \ln(x^2)$.
   a. For what real numbers x is f defined?
   b. Find the zeros of f.
   c. Write an equation for the line tangent to the graph of f at x = 1.

100. A particle moves along the x-axis so that at time t its position is given by $x(t) = t^3 - 6t^2 + 9t + 11$.
   a. What is the velocity of the particle at t = 0?
   b. During what time intervals is the particle moving to the left?
   c. What is the total distance traveled by the particle from t = 0 to t = 2?

101. Let f be the function defined for $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$ by $f(x) = x + \sin^2 x$.
   a. Find all values of x for which $f'(x) = 1$.
   b. Find the x-coordinates of <u>all</u> minimum points of f. Justify your answer.
   c. Find the x-coordinates of <u>all</u> inflection points of f. Justify your answer.

102. Let R be the shaded region between the graph of $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$ and the x-axis from x = 0 to x = 1.
   a. Find the area of R by setting up and integrating a definite integral.
   b. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region R about the x-axis.
   c. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region R about the line x = 1.

103. At time t = 0, a jogger is running at a velocity of 300 meters per minute. The jogger is slowing down with a negative acceleration that is directly proportional to time t. This brings the jogger to a stop in 10 minutes.
   a. Write an expression for the velocity of the jogger at time t.
   b. What is the total distance traveled by the jogger in that 10-minute interval?

104. A particle moves along the x-axis so that, at any time $t \ge 0$, its acceleration is given by $a(t) = 6t + 6$. At time t = 0, the velocity of the particle is -9, and its position is -27.
   a. Find v(t), the velocity of the particle at any time $t \ge 0$.
   b. For what values of $t \ge 0$ is the particle moving to the right?
   c. Find x(t), the position of the particle at any time $t \ge 0$.

105. Let f be the function defined by $f(x) = \frac{x + \sin x}{\cos x}$ for $\frac{-\pi}{2} < x < \frac{\pi}{2}$.
   a. State whether f is an even function or an odd function. Justify your answer.
   b. Find $f'(x)$.
   c. Write an equation of the line tangent to the graph of f at the point where x = 0.

106. Let R be the region enclosed by the x-axis, the y-axis, the line x = 2, and the $y = 2e^{x} + 3x$.

a. Find the area of R by setting up and evaluating the definite integral. Your work must include an antiderivative.

b. Find the volume of the solid generated by revolving R about the y-axis by setting up and evaluating a definite integral. Your work must include an antiderivative.

107. A function f is continuous on the closed interval [-3,3] such that f(-3) = 4 and f(3) = 1. The functions f′ and f″ have the properties given in the table below.

x	f′(x)	f″(x)
-3 < x < -1	positive	positive
x = -1	fails to exist	fails to exist
-1 < x < 1	negative	positive
x = 1	zero	zero
1 < x < 3	negative	negative

a. What are the x-coordinates of all absolute maximum and minimum points of f on the interval [-3,3]? Justify your answer.

b. What are the x-coordinates of all points of inflection on the interval [-3,3]? Justify your answer.

c. On the axes provided, sketch a graph that satisfies the given properties of f.

108. The volume V of a cone is increasing at the rate of 28 π cubic inches per second. At the instant when the radius r on the cone is 3 units, its volume is 12 π cubic units and the radius is increasing at $\frac{1}{2}$ unit per second.

a. At the instant when the radius of the cone is 3 units, what is the rate of change of the area of its base?

b. At the instant when the radius of the cone is 3 units, what is the rate of change of its height h?

c. At the instant when the radius of the cone is 3 units, what is the instantaneous rate of change of The area of its base with respect to its height h?

109. Let f be the function given by $f(x) = \frac{2x-5}{x^2-4}$.

a. Find the domain of f.

b. Write an equation for each vertical and each horizontal asymptote for the graph of f.

c. Find f′ (x).

d. Write an equation for the line tangent to the graph of f at the point (0,f(0)).

110. A particle moves along the x-axis with acceleration given by $a(t) = \cos(t)$ for $t \geq 0$. At $t = 0$ the velocity $v(t)$ of the particle is 2 and the position $x(t)$ is 5.

a. Write an expression for the velocity $v(t)$ of the particle.
b. Write an expression for the position $x(t)$.
c. For what values of t is the particle moving to the right? Justify your answer.
d. Find the total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$..

111. Let R be the region enclosed by the graphs of $y = e^{-x}$, $y = e^{x}$ and $x = \ln 4$.

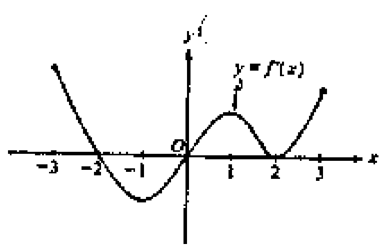
a. Find the area of R by setting up and evaluating a definite integral.
b. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume generated when the region R is revolved about the x-axis.
c. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume generated when the region R is revolved about the y-axis.

112. Let $f(x) = (14\pi)x^2$ and $g(x) = k^2 \sin\frac{\pi x}{2k}$ for $k > 0$.

a. Find the average value of f on [1,4].
b. For what value of k will the average value of g on [0,k] be equal to the average value of f on [1,4]?

113. A balloon in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute.

a. At this instant, what is the height of the cylinder?
b. At this instant, how fast is the height of the cylinder increasing?



114. The figure above shows the graph of f', the derivative of a function f.
The domain of the function f is the set of all x such that $-3 \leq x \leq 3$.

a. For what values of x, $-3 < x < 3$, does f have a relative maximum? A relative minimum? Justify your answer.
b. For what values of x is the graph of f concave up? Justify your answer.
c. Use the information found in parts (a) and (b) and the fact that $f(-3) = 0$ to sketch a possible graph of f on the axes provided.

115. Let f be the function defined by $f(x) = 7 - 15x + 9x^2 - x^3$ for all real numbers x.

a. Find the zeros of f.

b. Write an equation of the line tangent to the graph of f at x = 2.

c. Find the x-coordinates of all points of inflection of f. Justify your answer.

116. Let f be the function given by $f(x) = \frac{9x^2 - 36}{x^2 - 9}$.

a. Describe the symmetry of the graph of f.

b. Write an equation for each vertical and each horizontal asymptote of f.

c. Find the intervals on which f is increasing.

d. Using the results found in parts (a), (b), and (c), sketch the graph of f.

117. A particle moving along the x-axis so that at any time $t \geq 1$ its acceleration is given by $a(t) = \frac{1}{t}$. At time t = 1, the velocity of the particle is v(1) = -2 and its position is x(1) = 4.

a. Find the velocity v(t) for $t \geq 1$.

b. Find the position x(t) for $t \geq 1$.

c. What is the position of the particle when it is farthest to the left?

118. Let f be the function defined as follows:

$$f(x) = \begin{cases} |x - 1| + 2, & \text{for } x < 1 \\ ax^2 + bx, & \text{for } x \geq 1, \text{ where a and b are constants.} \end{cases}$$

a. If a = 2 and b = 3, is f continuous of all x? Justify your answer.

b. Describe all values of a and b for which f is a continuous function.

c. For what values of a and b if f both continuous and differentiable?

119. Let A(x) be the area of the rectangle inscribed under the curve $y = e^{-2x^2}$ with vertices at (-x,0) and (x,0), $x \geq 0$.

a. Find A(1).

b. What is the greatest value of A(x)? Justify your answer.

c. What is the average value of A(x) on the interval $0 \leq x \leq 2$?

120. The region enclosed by the graphs of $y = \tan^2 x$, $y = \frac{1}{2}\sec^2 x$, and the y-axis.

a. Find the area of the region R.

b. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region about the x-axis.

121. A particle moves along the x-axis so that its acceleration at any time t is given by $a(t) = 6t - 18$. At time $t = 0$ the velocity of the particle is $v(0) = 24$, and at time $t = 1$, its position is $x(1) = 20$.

a. Write an expression for the velocity $v(t)$ of the particle at any time t.

b. For what values of t is the particle at rest?

c. Write an expression for the position $x(t)$ of the particle at any time t.

d. Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

122. Let $f(x) = \sqrt{1-\sin x}$.

a. What is the domain of f?

b. Find $f'(x)$.

c. What is the domain of f'?

d. Write an equation for the line tangent to the graph of f at $x = 0$.

123. Let R be the region enclosed by the graphs of $y = (64x)^{(1/4)}$ and $y = x$.

a. Find the volume of the solid generated when region R is revolved about the x-axis.

b. Set up, but do not integrate, an integral expression in terms of a single variable the volume of the solid generated when the region R is revolved about the y-axis.

124. Let f be the function given by $f(x) = 2\ln(x^2 + 3) - x$ with domain $-3 \leq x \leq 5$.

a. Find the x-coordinate of each relative maximum point and each relative minimum point of f. Justify your answer.

b. Find the x-coordinate of each inflection point of f.

c. Find the absolute maximum value of $f(x)$.

125. A trough is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

a. Find the volume of water in the trough when it is full.

b. What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?

c. What is the rate of change in the area of the surface of the water at the instant when the trough is $\frac{1}{4}$ full by volume?

126. Let f be a function such that $f(x) < 1$ and $f'(x) < 0$ for all x.

a. Suppose that $f(b) = 0$ and $a < b < c$. Write an expression involving integrals for the area of the region enclosed by the graph of f, the lines $x = a$ and $x = c$, and the x-axis.

b. Determine whether $g(x) = \frac{1}{f(x)-1}$ is increasing or decreasing. Justify your answer.

c. Let he be a differentiable function such that $h'(x) < 0$ for all x. Determine whether $F(x) = H(f(x))$ is increasing or decreasing. Justify your answer.

127. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

a. Find the domain of f.
b. Describe the symmetry, if any, of the graph of f.
c. Find $f'(x)$.
d. Find the slope of the line normal to the graph of f at x = 5.

128. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

a. Find the acceleration a(t) of the particle at any time t.
b. Find all values of t, $0 \leq t \leq 2$, for which the particle is at rest.
c. Find the position x(t) of the particle at any time t if x(0) = 0.

129. Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line x = 5.

a. Find the volume of the solid generated by revolving R about the x-axis.
b. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the line x = -1.

130. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

a. Write an equation of the horizontal asymptote for the graph of f.
b. Find the x-coordinate of each critical point of f. For each such x, determine whether f(x) is a relative maximum, a relative minimum, or neither.
c. For what values of x is the graph of f concave down?
d. Using the results found in parts a, b, and c, sketch the graph of y = f(x) in the xy-plane provided.

131. Let R be the region in the first quadrant under the graph of $y = \dfrac{x}{x^2+2}$ for $0 \leq x \leq \sqrt{6}$.

a. Find the area of R.
b. If the line x = k divides R into two regions of equal area, what is the value of k?
c. What is the average value of $y = \dfrac{x}{x^2+2}$ on the interval $0 \leq x \leq \sqrt{6}$?

132. Let f be a differentiable function, defined for all real numbers x, with the following properties.

(i) $f'(x) = ax^2 + bx$
(ii) $f'(1) = 6$ and $f''(1) = 18$
(iii) $\int_1^2 f(x)\,dx = 18$.

Find f(x). Show your work.

133. Let f be the function given by $f(x) = x^3 - 7x + 6$.

a. Find the zeros of f.

b. Write an equation of the line tangent to the graph of f at $x = -1$.

c. Find the number c that satisfies the conclusion of the Mean Value Theorem for f on the closed interval [1,3].

134. Let R be the region in the first quadrant enclosed by the graph of $y = \sqrt{6x+4}$, the line $y = 2x$, and the y-axis.

a. Find the area of R.

b. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the x-axis.

c. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

135. A particle moves along the x-axis in such a way that its acceleration at time t for t = 0 is given by $a(t) = 4\cos(2t)$. At time t = 0, the velocity of the particle is $v(0) = 1$ and its position is $x(0) = 0$.

a. Write an equation for the velocity $v(t)$ of the particle.

b. Write an equation for the position $x(t)$ of the particle.

c. For what values of t, $0 \le t \le \pi$, is the particle at rest?

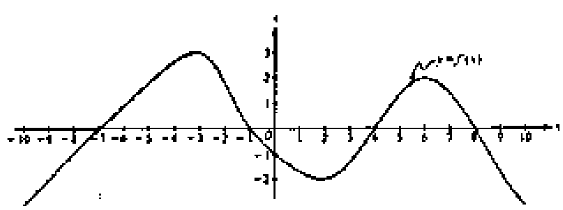
136. Let f be the function given by $f(x) = \dfrac{x}{\sqrt{x^2 - 4}}$.

a. Find the domain of f.

b. Write an equation for each vertical asymptote to the graph of f.

c. Write an equation for each horizontal asymptote to the graph of f.

d. Find $f'(x)$.



137. The figure above shows the graph of f′, the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \le x \le 10$.

a. For what values of x does the graph of f have a horizontal tangent?

b. For what values of x in the interval (-10, 10) does f have a relative maximum?

c. For what values of x is the graph of f concave downward?

138. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $\frac{dy}{dt} = ky$, where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

a. Write an equation for y, the amount of oil remaining in the well at any time t.
b. At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil Remaining?
c. In order not to lose money, at what time t should oil no longer be pumped from the well?

139. A particle, initially at rest, moves along the x-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t^2 - 4$. The position of the particle when t=1 is $x(1) = 3$.

a. Find the values of t for which the particle is at rest.
b. Write an expression for the position x(t) of the particle at any time $t \geq 0$.
c. Find the total distance traveled by the particle from t = 0 to t = 2.

140. Let f be the function given by $\ln\frac{x}{x-1}$.

a. What is the domain of f?
b. Find the value of the derivative of f at x = -1.
c. Write an expression for $f^{-1}(x)$, where f^{-1} denotes the inverse function of f.

141. Let R be the region enclosed by the graphs of $y = e^x$, $y = (x - 1)^2$, and the line x = 1.

a. Find the area of R.
b. Find the volume of the solid generated when R is revolved about the x-axis.
c. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

142. The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second.

a. At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?
b. At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?
c. At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

143. Let f be the function defined by $f(x) = \sin^2 x - \sin(x)$ for $0 \leq x \leq \frac{3\pi}{2}$.

a. Find the x-intercepts of the graph of f.
b. Find the intervals on which f is increasing.
c. Find the absolute maximum value and the absolute minimum value of f. Justify your answer.

144. Let f be the function that is given by $f(x) = \frac{ax+b}{x^2 - c}$ and that has the following properties.

i) The graph of f is symmetric with respect to the y-axis.

ii) $\lim_{x \to 2+} f(x) = +\infty$

iii) f′ (1) = -2

a. Determine the values of a, b, and c.

b. Write an equation for each vertical and each horizontal asymptote of the graph of f.

c. Sketch the graph of f in the xy-plane.

145. Let f be the function that is defined for all real numbers x and that has the following properties.

i) f″(x) = 24x - 18

ii) f′ (1) = -6

iii) f(2) = 0

a. Find each x such that the line tangent to the graph of f at (x,f(x)) is horizontal.

b. Write an expression for f(x).

c. Find the average value of f on the interval $1 \le x \le 3$

146. Let R be the region between the graphs of y = 1 + sin(πx) and $y = x^2$ from x = 0 to x = 1.

a. Find the area of R.

b. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the x-axis.

c. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

147. Let f be the function defined by $f(x) = (1 + \tan x)^{1.5}$ for $\frac{\pi}{4} < x < \frac{\pi}{2}$.

a. Write an equation for the line tangent to the graph of f at the point where x = 0.

b. Using the equation found in part a, approximate f(0.02).

c. Let f^{-1} denote the inverse function of f. Write an expression that gives $f^{-1}(x)$ for all x in the domain of f^{-1}.

148. Let f be the function given by $f(x) = \frac{|x|-2}{x-2}$.

a. Find all the zeros of f.

b. Find f′ (1).

c. Find f ′(-1).

d. Find the range of f.

149. Let f be a function that is even and continuous on the closed interval [-3,3]. The function f and its derivatives have the properties indicated in the table below.

x	0	0<x<1	1	1<x<2	2	2<x<3
f(x)	1	positive	0	negative	-1	negative
f′(x)	∅	negative	0	negative	∅	positive
f″(x)	∅	positive	0	negative	∅	negative

a. Find the x-coordinate of each point at which f attains an absolute maximum value or an absolute minimum value. For each coordinate you give, state whether f attains an absolute maximum or an absolute minimum.
b. Find the x-coordinate of each point of inflection on the graph of f. Justify your answer.
c. In the xy-plane provided, sketch the graph of a function with all the given characteristics of f.

150. A tightrope is stretched 30 feet above the ground between the Jay and Tee buildings which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A.
a. How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)
b. How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)
c. How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee Building when she is 10 feet from point B? (Indicate units.)

151. Let f be the function defined by $f(x) = 3x^5 - 5x^3 + 2$.
a. On what interval is f increasing?
b. On what intervals is the graph of f concave upward?
c. Write an equation on each horizontal tangent line to the graph of f.

152. A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$, is given by $v(t) = 3(t-1)(t-3)$. At time t = 2, the position of the particle is $x(2)=0$.
a. Find the minimum acceleration of the particle.
b. Find the total distance traveled by the particle.
c. Find the average velocity of the particle over the interval $0 \le t \le 5$.

153. Let f be the function given by $f(x) = \ln \left| \frac{x}{1+x^2} \right|$.
a. Find the domain of f.
b. Determine whether f is an even function, an odd function, or neither. Justify your conclusion.
c. At what values of x does f have a relative maximum or a relative minimum? For each such x, use the first derivative test to determine whether f(x) is a relative maximum or a relative minimum.
d. Find the range of f.

154. Consider the curve defined by the equation $y + \cos(y) = x + 1$ for $0 \le y \le 2\pi$.
a. Find dy/dx in terms of y.
b. Write an equation for each vertical tangent to the curve.
c. Find $\frac{d^2y}{dx^2}$ in terms of y.

155. Let f be the function given by $f(x) = e^{-x}$, and let g be the function given by $g(x) = kx$, where k is the nonzero constant such that the graph of f is tangent to the graph of g.

a. Find the x-coordinate of the point of tangency and the value of k.

b. Let R be the region enclosed by the y-axis and the graph of f and g. Using the results found in part a, determine the area of R.

c. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated by revolving the region R, given in part b, about the x-axis.

156. At time t, $t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2.

a. Find the radius of the sphere as a function of t.

b. At what time t will the volume of the sphere be 27 times its volume at $t = 0$?

<u>Scientific Calculators required</u>

157. Let f be the function given by $f(x)= x^3 -5x^2 + 3x + k$, where k is a constant.

a. On what interval is f increasing?

b. On what intervals is the graph of f concave downward?

c. Find the value of k for which f has 11 as its relative minimum.

158. A particle moves on the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = 2te^{-t}$.

a. Find the acceleration of the particle at $t = 0$.

b. Find the velocity of the particle when its acceleration is 0.

c. Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

159. Consider the curve $y^2 = 4 + x$ and chord AB joining points A(-4,0) and B(0,2) on the curve.

Find the x- and y-coordinates of the points on the curve where the tangent line is parallel to chord AB.

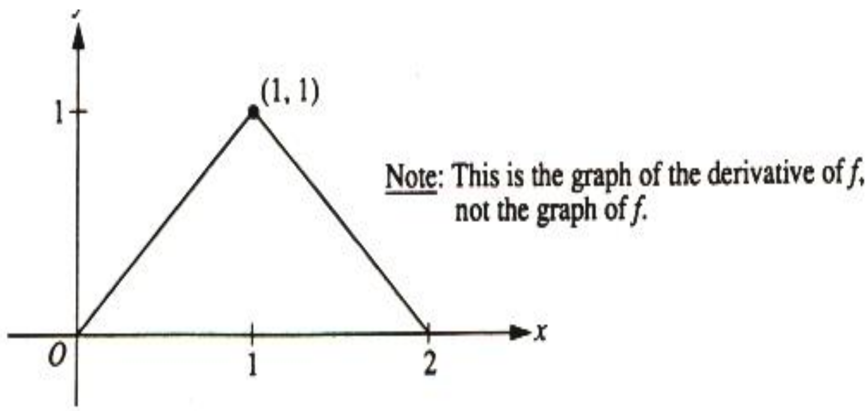
b. Find the area of the region R enclosed by the curve and the chord AB.

c. Find the volume of the solid generated when the region R, defined in part b, is revolved about the x-axis.

160. Let f be the function defined by $f(x) = \ln (2 + \sin(x)$ for $\pi \leq x \leq 2\pi$.

a. Find the absolute maximum value and the absolute minimum value of f. Show the analysis that leads to your conclusion.

b. Find the x-coordinate of each inflection point on the graph of f. Justify your answer.



The figure above shows the graph of f′ the derivative of a function f The domain of f is the

161. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all x such that $0 < x < 2$.
   a. Write an expression of $f'(x)$ in terms of x.
   b. Given that $f(1) = 0$, write an expression for $f(x)$ in terms of x.
   c. Sketch the graph of $y = f(x)$.

162. Let P(t) represent the number of wolves in a population at time t years, when $t \geq 0$. The population P(t) is increasing at a rate proportional to 800 - P(t), where the constant of proportionality is k.
   a. If P(0) = 500, find P(t) in terms of t and k.
   b. If P(2) = 700, find k.
   c. Find the limit as t approaches infinity of P(t).

Scientific Calculators required

163. Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.
   a. Write an equation of the line tangent to the graph of f at the point (2, -28)
   b. Find the absolute minimum value of f. Show the analysis that leads to your conclusion.
   c. Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

164. Let R be the region enclosed by the graphs of $y = e^x$, $y = x$, and the lines $x = 0$ and $x = 4$.
   a. Find the area of R.
   b. Find the volume of the solid generated when R is revolved about the x-axis.
   c. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

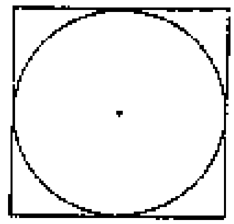
165. Consider the curve defined by $x^2 + xy + y^2 = 27$.
   a. Write an expression for the slope of the curve at any point (x,y).
   b. Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel.
   c. Find the points on the curve where the lines tangent to the curve are vertical.

166. A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t \ln t - t$.

a. Write an expression for the acceleration of the particle.

b. For what values of t is the particle moving to the right?

c. What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.



167. A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency.

a. Find the rate at which the <u>perimeter</u> of the square is increasing. Indicate units of measure.

b. At the instant when the area of the circle is 25π square inches, find the rate of increase in the <u>area enclosed between</u> the circle and the square. Indicate units of measure.

168. Let F(x) = $\int_0^x \sin t^2 \, dt$ for $0 \le x \le 3$.

a. Use the Trapezoidal Rule with four equal subdivisions of the closed interval [0, 1] to approximate F(1).

b. On what intervals is F increasing?

c. If the average rate of change of F on the closed interval [1, 3] is k, find the $\int_1^3 \sin t^2 \, dt$ in terms of k.

<u>Graphing Calculators Required on all 6 questions</u>

169. Let f be the function given by $f(x) = \dfrac{2x}{\sqrt{x^2 + x + 1}}$.

a. Find the domain of f. Justify your answer.

b. In the viewing window [-5,5] x [-3,3], sketch the graph of f.

c. Write an equation for each horizontal asymptote of the graph of f.

d. Find the range of f. Use $f'(x)$ to justify your answer.

170. A particle moves along the y-axis so that its velocity at any time $t \ge 0$ is given by $v(t) = t\cos(t)$. At time $t = 0$, the position of the particle is $y = 3$.

a. For what values of t, $0 \le t \le 5$, is the particle moving upward?

b. Write an expression for the acceleration of the particle in terms of t.

c. Write an expression for the position y(t) of the particle.

d. For $t > 0$, find the position of the particle the first time the velocity of the particle is zero.

171. Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

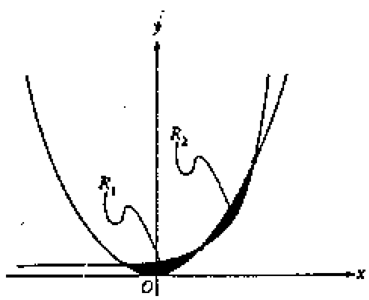
a. Find $\frac{dy}{dx}$.

b. Write an equation for the line tangent to the curve at the point (4,-1).

c. There is a number k so that the point (4.2, k) is on the curve. Using the tangent line found in part b, approximate the value of k.

d. Write an equation that can be solved to find the actual value of k so that the point (4.2, k) is on the curve.

e. Solve the equation found in part d for the value of k.



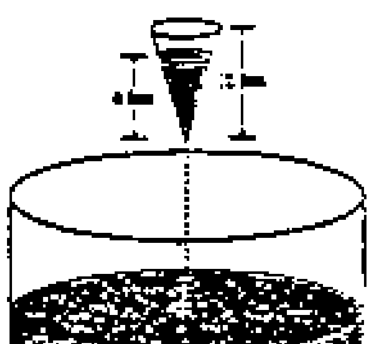
172. The shaded regions R_1 and R_2 shown above are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.

a. Find the x- and y-coordinates of the three points of intersection of the graphs of f and g.

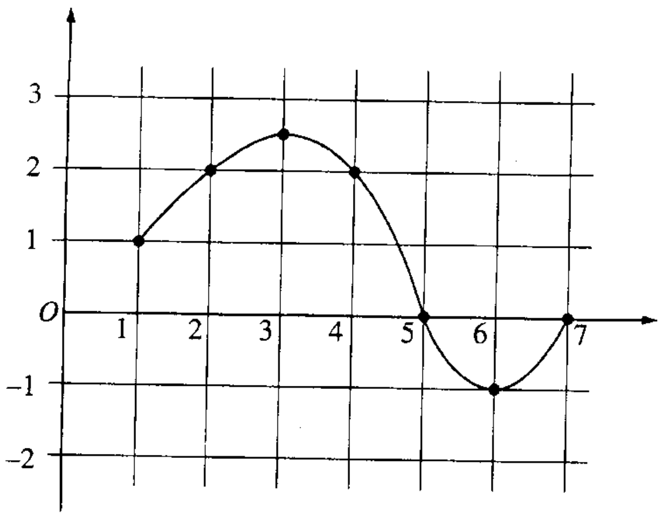
b. Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of f and g. Do not evaluate.

c. Without using absolute value, set up an expression involving one or more integrals that gives the volume of the solid generated by revolving the region R_1 about the line y = 5. Do not evaluate.

173. As shown in the figure below, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of (h – 12) feet per minute.



a. Write an expression for the volume of water in the conical tank as a function of h.
b. At what rate is the volume of water in the conical tank changing when h = 3? Indicate the units of measure.
c. Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when h = 3? Indicate units of measure.



174. The graph of a differentiable function f on the closed interval [1,7] is shown.

Let $h(x) = \int f(t)\,dt$ for $1 \le x \le 7$.

a. Find h(1).
b. Find h′ (4).
c. On what interval or intervals is the graph of h concave upward? Justify your answer.
d. Find the value of x at which h has its minimum on the closed interval [1,7]. Justify your answer.

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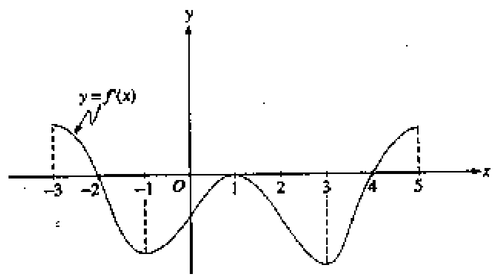


Note: This is the graph of the derivative of f, not the graph of f.

175. The figure above shows the graph of f′, the derivative of a function f. The domain of f is the set of all real numbers x such that -3 < x < 5.

a. For what values of x does f have a relative maximum? Why?
b. For what values of x does f have a relative minimum? Why?
c. On what intervals is the graph of f concave upward? Use f′ to justify your answer.
d. Suppose that f(1) = 0. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function f on the open interval 0 < x < 2.

176. Let R be the region in the first quadrant under the graph of y = $\frac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

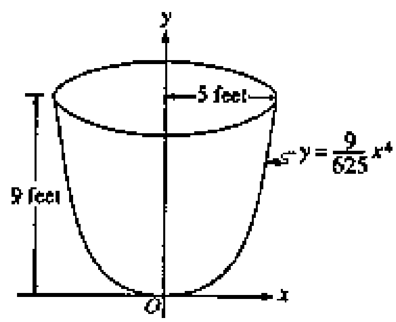
a. Find the area of R.
b. If the line x = k divides the region R into two regions of equal area, what it is the value of k?
c. Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.

177. The rate of consumption of cola in the United States is given by S(t) = Ce^{kt}, where S is measured in billions of gallons per year and t is measured in years from the beginning of 1980.

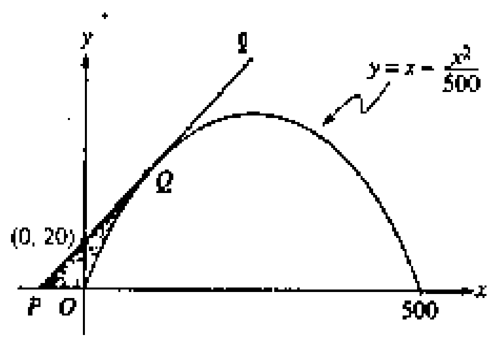
a. The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find *C* and k.
b. Find the average rate of consumption of cola over the 10-year time period beginning January 1, 1983. Indicate units of measure.
c. Use the trapezoidal rule with four equal subdivisions to estimate the integral from 5 to 7 of S(t) dt.
d. Using correct units, explain the meaning of the integral from 5 to 7 of S(t) dt in terms of cola consumption.

178. This problem deals with functions defined by f(x) = x + b sin x, where b is positive and constant and [- 2π, 2π].

a. Sketch the graphs of two of these functions, y = x + sin(x) and y = x + 3 sin(x), as indicated below.
b. Find the x-coordinates of all points, [-2π, 2π], where the line y = x + b is tangent to the graph of f(x) = x + b sin(x).
c. Are the points of tangency described in part (b) relative maximum points of f ? Why?
d. For all values of b > 0, show that all inflection points of the graph of f lie on the line y = x.



179. An oil storage tank has the shape shown above, obtained by revolving the curve $y = \frac{9}{625}x^4$ from $x = 0$ to $x = 5$ about the y-axis, where x and y are measured in feet. Oil flows into the tank at the constant rate of 8 cubic feet per minute.
   a. Find the volume of the tank. Indicate units of measure.
   b. To the nearest minute, how long would it take to fill the tank if the tank was empty initially?
   c. Let h be the depth, in feet, of oil in the tank. How fast is the depth of the oil in the tank increasing when h = 4? Indicate units of measure.



180. Let l be tangent to the graph of $y = x - \frac{x^2}{500}$ at the point Q, as shown in the figure above.
   a. Find the x-coordinate of point Q.
   b. Write an equation for line ℓ.
   c. Suppose the graph of $y = x - \frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along the line *l* shine on any part of the tree? Show the work that leads to your conclusion.

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181. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position x(t) is 5 for t = 2.
   a. Write a polynomial expression for the position of the particle at any time $t \geq 0$.
   b. For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average

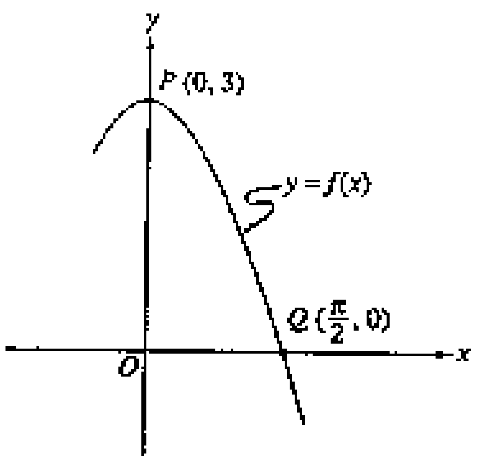
velocity on the closed interval [0,3]?

c. Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

182. Let f be the function given by $f(x) = 3\cos(x)$. The graph of f crosses the y-axis at point P and the x-axis at point Q.

a. Write an equation for the line passing through points P and Q.

b. Write an equation for the line tangent to the graph of f at point Q. Show the analysis that leads



to your equation.

c. Find the x-coordinate of the point on the graph of f, between points P and Q, at which the line tangent to the graph of f is parallel to line PQ.

d. Let R be the region in the first quadrant bounded by the graph of f and line segment PQ. Write an integral expression for the volume of the solid generated by revolving the region R about the x-axis. Do not evaluate.

183. Let f be the function given by f(x) = $\sqrt{x-3}$.

a. Sketch the graph of f and shade the region R enclosed by the graph of f, the x-axis, and the vertical line x = 6.

b. Find the area of the region R described in part (a).

c. Rather than using the line x = 6 as in part (a), consider the line x = w, where w can be any number greater than 3. Let A(w) be the area of the region enclosed by the graph of f, the x-axis, and the vertical line x = w. Write an integral expression for A(w).

d. Let A(w) be as described in part ©. Find the rate of change of A with respect to w when w = 6.

184. Let f be the function given by f(x) = $x^3 - 6x^2 + p$, where p is an arbitrary constant.

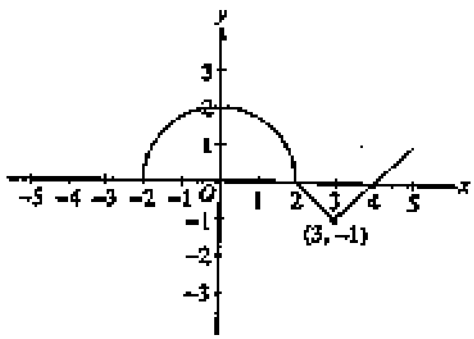
a. Write an expression for f′(x) and use it to find the relative maximum and minimum values of f in terms of p. Show the analysis that leads to your conclusion.

b. For what values of the constant p does f have 3 distinct real roots?

c. Find the value of p such that the average value of f over the closed interval [-1,2] is 1.

185. The graph of a function f consists of a semicircle and two line segments as shown below. Let g be the function given by $\int_0^x f(t)dt$.

a. Find g(3).
b. Find all values of x on the open interval (-2,5) at which g has a relative maximum. Justify your answer.
c. Write an equation for the line tangent to the graph of g at x = 3.
d. Find the x-coordinate of each point of inflection of the graph of g on the open interval (-2,5). Justify your answer.



186. Let v(t) be the velocity, in feet per second, of a skydiver at time t seconds, t $\geq$ 0. After her parachute opens, her velocity satisfies the differential equation $\frac{dy}{dx}$ = -2v - 32, with initial condition v(0) = -50.

a. Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.
b. Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.
c. It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

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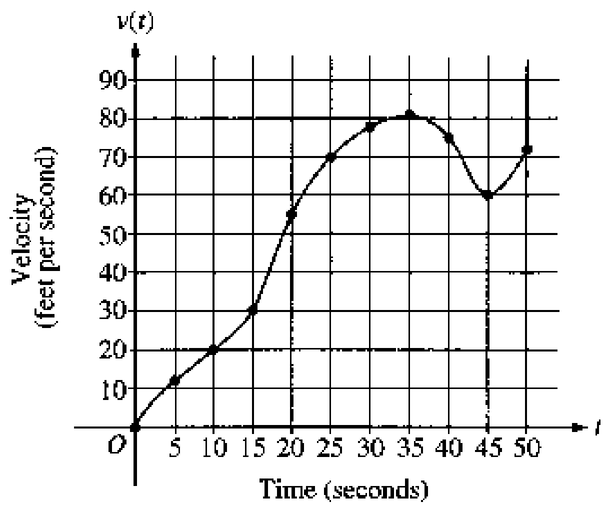
187. Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line x = 4.
   a. Find the area of the region R.
   b. Find the value of h such that the vertical line x = h divides the region R into two regions of equal area.
   c. Find the volume of the solid generated when R is revolved about the x-axis.
   d. The vertical line x = k divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

188. Let f be the function given by $f(x) = 2xe^{2x}$.
   a. Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.
   b. Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   c. What is the range of f?
   d. Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same of all nonzero values of b.



t (seconds)	v(t) (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

189. The graph of the velocity v(t), in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for v(t), at 5 second intervals of time t, is shown to the right of the graph.
   a. During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
   b. Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.
   c. Find one approximation for the acceleration of the car in ft/sec^2, at t = 40. Show the computations you used to arrive at your answer.
   d. Approximate the integral from 0 to 50 of v(t) dt with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

190. Let f be the function with f(1) = 4 such that for all points (x,y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

a. Find the slope of the graph of f at the point where x = 1.
b. Write an equation for the line tangent to the graph of f at x = 1 and use it to approximate f(1.2).
c. Find f(x) by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition f(1) = 4.
d. Use your solution from part © to find f(1.2).

191. The temperature outside a house during a 24-hour period is given by $F(t) = 80 + 10\cos\frac{\pi t}{12}$, $0 \le t \le 24$, where F(t) is measured in degrees Fahrenheit and t is measured in hours.

a. Sketch the graph of F on the grid provided.
b. Find the average temperature, to the nearest degree Fahrenheit, between t = 6 and t = 14.
c. An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?
d. The cost of cooling the house accumulates at the rate of $0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit.
e. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

192. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

a. Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.
b. Write an equation of each horizontal tangent line to the curve.
c. The line through the origin with slope -1 is tangent to the curve at point P. Find the x- and y-coordinates of point P.

1999 *Graphing Calculators Required on all 6 questions*

193. A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \ge 0$.

a. In which direction (up or down) is the particle moving at time t = 1.5? Why?
b. Find the acceleration of the particle at time t = 1.5. Is the velocity of the particle increasing at t = 1.5? Why or why not?
c. Given that y(t) is the position of the particle at time t and the y(0) = 3, Find y(2).
d. Find the total distance traveled by the particle from t = 0 to t = 2.

194. The shaded region, R, is bounded by the graph of $y = x^2$ and the line y = 4.

a. Find the area of R.
b. Find the volume of the solid generated by revolving R about the x-axis.
c. There exists a number k, k > 4, such that when R is revolved about the line y = k, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

t (hours)	$R(t)$ (gallons per hour
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

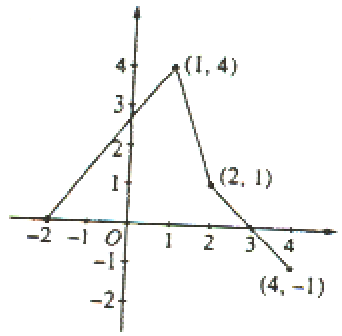
195. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table shows the rate as measured every 3 hours for a 24-hour period.

a. Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate from 0 to 24 of R(t) dt. Using correct units, explain the meaning of your answer in terms of water flow.
b. Is there some time t, 0 < t < 24, such that R′(t) = 0? Justify your answer.
c. The rate of water flow R(t) can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use Q(t) to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

196. Suppose that the function f has a continuous second derivative for all x, and that f(0) = 2, f ′(0) = -3, and f ″(0) = 0. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.
a. Write an equation of the line tangent to the graph of f at the point where x = 0.
b. Is there sufficient information to determine whether or not the graph of f has a point of inflection where x = 0? Explain your answer.
c. Given that g(0) = 4, write an equation of the line tangent to the graph of g at the point where x = 0.
d. Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at x = 0? Justify your answer.

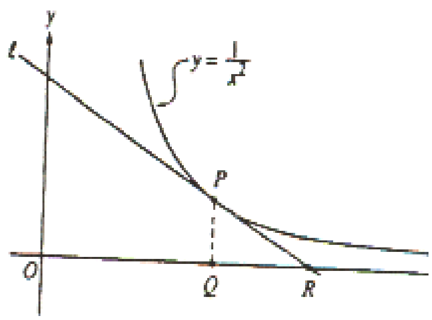
197. The graph of the function f, consisting of three line segments, is given below. Let

$g(x) = \int_1^x f(t)dt$



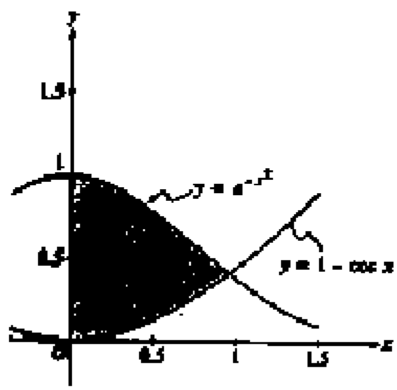
a. Compute g(4) and g
b. Find the instantaneous rate of change of g, with respect to x, at x = 1.
c. Find the absolute minimum value of g on the closed interval [-2,4]. Justify your answer.
d. The second derivative of g is not defined at x = 1 and x = 2. How many of these values are x-coordinates of points of inflection of the graph of f? Justify your answer.

198. In the figure below, line l is tangent to the graph of $y = \frac{1}{x^2}$ at point P, with coordinates $(w, \frac{1}{w^2})$, where w > 0. Point Q has coordinates (w,0). Line l crosses the x-axis at the point R, with coordinates (k, 0).

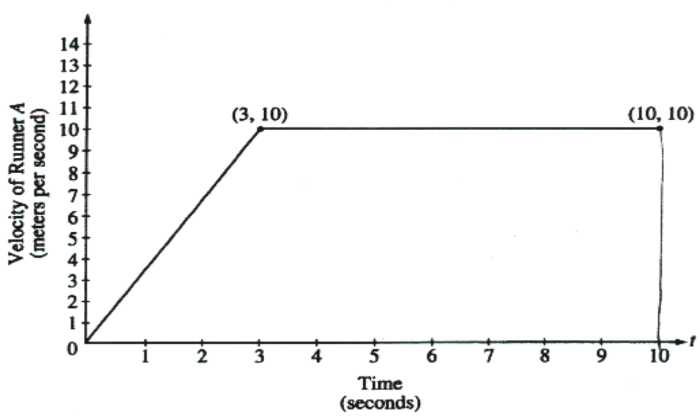


a. Find the value of k when w = 3.
b. For all w > 0, find k in terms of w.
c. Suppose that w is increasing at the constant rate of 7 units per second. When w = 5, what is the rate of change of k with respect to time?
d. Suppose that w is increasing at the constant rate of 7 units per second. When w = 5, what is the rate of change of the area of triangle PQR with respect to time? Determine whether the area is increasing or decreasing at this instant.

2000 *Graphing Calculator on 1st 3 questions only*

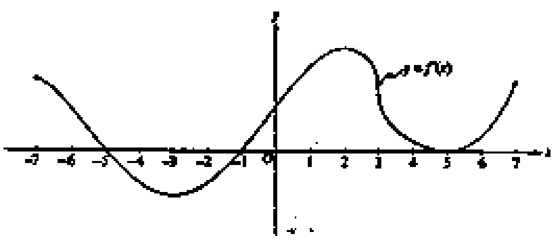


199. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos(x)$, and the y-axis.
   a. Find the area of the region R.
   b. Find the volume of the solid generated when the region R is revolved about the x-axis.
   c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



200. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph below, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \frac{24t}{2t+3}$.

a. Find the velocity of Runner A and the velocity of Runner B at time t = 2 seconds. Indicate units of measure.
b. Find the acceleration of Runner A and the acceleration of Runner B at time t = 2 seconds. Indicate units of measure.
c. Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.



201. The figure above shows the graph of f f', the derivative of the function f, for $-7 \le x \le 7$. The Graph of f' has horizontal tangent lines at x = -3, x = 2, and x = 5, and a vertical tangent line at x = 3.

a. Find all values of x, for $-7 \le x \le 7$, at which f attains a relative minimum. Justify your answer.
b. Find all values of x, for $-7 \le x \le 7$, at which f attains a relative maximum. Justify your answer.
c. Find all values of x, for $-7 \le x \le 7$, at which $f''(x) < 0$.
d. At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

202. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time t = 0, the tank contains 30 gallons of water.

a. How many gallons of water leak out of the tank from time t= 0 to t = 3 minutes?
b. How many gallons of water are in the tank at time t = 3 minutes?
c. Write an expression for A(t), the total number of gallons of water in the tank at time t.
d. At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

203. Consider the curve given by $xy^2 - x^3y = 6$.

a. Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

b. Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

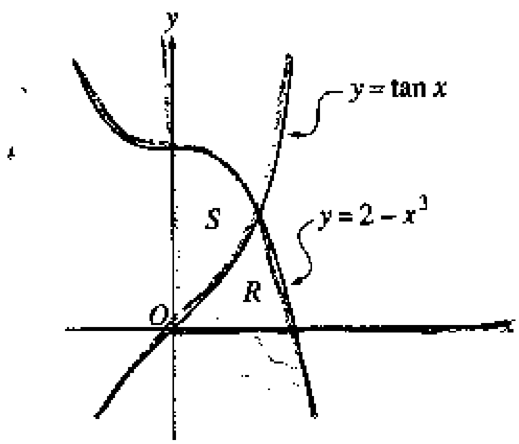
c. Find the x-coordinate of each point on the curve where the tangent line is vertical.

204. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

a. Find a solution y = f(x) to the differential equation satisfying $f(0) = \frac{1}{2}$.

b. Find the domain and range of the function f found in part (a).

2001 *Graphing Calculator on 1st 3 questions only*



205. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

a. Find the area of R.

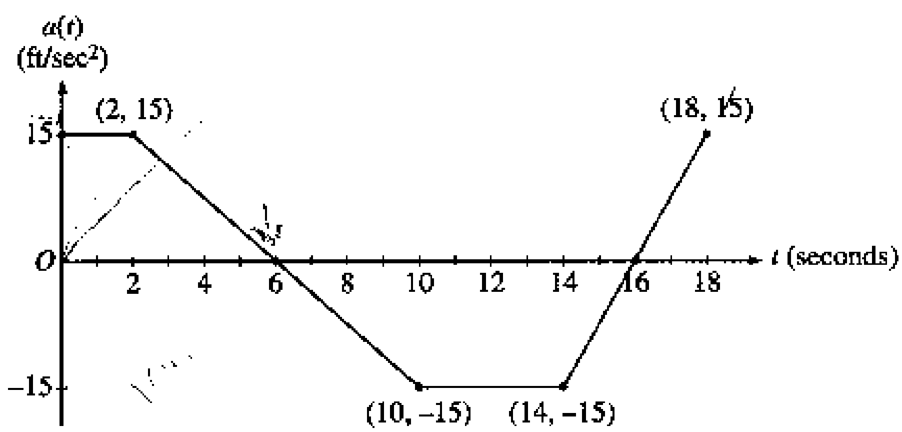
b. Find the area of S.

c. Find the volume of the solid generated when S is revolved about the x-axis.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

206. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

a. Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.
b. Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.
c. A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and P(t) is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.
d. Use the function P defined in part c to find the average value, in degrees Celsius, of P(t) over the time interval $0 \le t \le 15$ days.



207. A car traveling on a straight road with velocity 55 ft/sec at time t = 0. For $0 \le t \le 18$ seconds, the car's acceleration a(t), in ft/sec^2, is the piecewise linear function defined by the graph above.

a. Is the velocity of the car increasing at t = 2 seconds? Why or why not?
b. At what time in the interval $0 \le t \le 18$, other than t = 0, is the velocity of the car 55 ft/sec? Why?
c. On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
d. At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

208. Let h be a function defined for all $x \neq 0$ such that h(4) = -3 and the derivative of h is given by

$h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.

a. Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

b. On what intervals, if any, is the graph of h concave up? Justify your answer.

c. Write an equation for the line tangent to the graph of h at x = 4.

d. Does the line tangent to the graph of h at x = 4 lie above or below the graph of h for x > 4? Why?

209. A cubic polynomial function f is defined by $f(x) = 4x^3 + ax^2 + bx + k$ where a, b, and k are constants. The function f has a local minimum at x = -1, and the graph of f has a point of inflection at x = -2.

a. Find the values of a and b.

b. If $\int_0^1 f(x)dx = 32$, what is the value of k?

210. The function f is differentiable for all real numbers. The point $(3, \frac{1}{4})$ is on the graph of y = f(x), and the slope at each point (x,y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

a. Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $(3, \frac{1}{4})$.

b. Find y = f(x) by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

2002 *Graphing Calculator on 1st 3 questions only*

211. Let *f* and *g* be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

a. Find the area of the region enclosed by the graphs of *f* and *g* between x = ½ and x = 1.

b. Find the volume of the solid generated when the region enclosed by the graphs of *f* and *g* between x = ½ and x = 1 is revolved about the line y = 4.

c. Let *h* be the function given by *h(x) = f(x) – g(x)*. Find the absolute minimum value of *h(x)* on the closed interval $\frac{1}{2} \leq x \leq 1$, and find the absolute maximum value of *h(x)* on the closed interval $\frac{1}{2} \leq x \leq 1$. Show the analysis that leads to your answers.

212. The rate at which people enter an amusement park on a given day is modeled by the function *E* defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function *L* defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both *E(t)* and *L(t)* are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \leqslant t \leqslant 23$, the hours during which the park is open. At time t =9, there are no people in the park.

a. How many people have entered the park by 5:00 P.M. (t = 17)? Round your answer to the nearest whole number.
b. The price of admission to the park is $15 until 5:00 P.M. (t = 17). After 5:00 P.M., the price of admission to the park is $11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
c. Let $H(t) = \int_9^t (E(x) - L(x))dx$ for $9 \leqslant t \leqslant 23$. The value of *H*(17) to the nearest whole number is 3725.

   Find the value of *H′*(17), and explain the meaning of *H*(17) and *H′*(17) in the context of the amusement park.
d. At what time t, for $9 \leqslant t \leqslant 23$, does the model predict that the number of people in the park is a maximum?

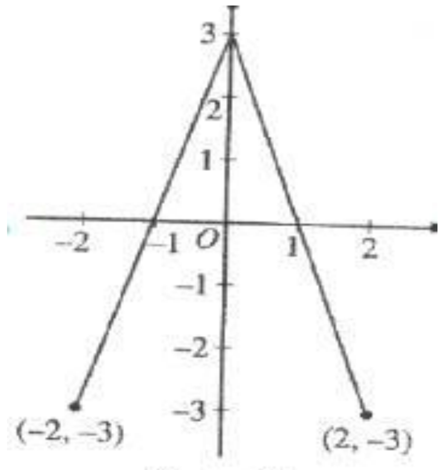
213. An object moves along the x-axis with initial position *x(0)* = 2. The velocity of the object at time $t \geqslant 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

a. What is the acceleration of the object at time t = 4?
b. Consider the following two statements.

   Statement I: For 3 < x < 4.5, the velocity of the object is decreasing.

   Statement II: For 3 < x < 4.5, the speed of the object is increasing.

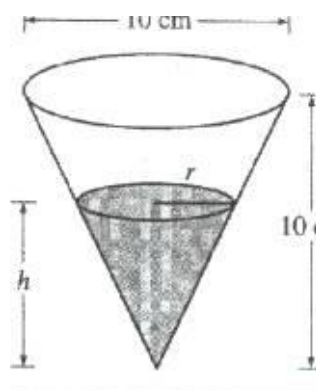
   Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.
c. What is the total distance traveled by the object over the time interval $0 \leqslant t \leqslant 4$?
d. What is the position of the object at time t = 4?



Graph of f

214. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

a. Find $g(-1)$, $g'(-1)$, and $g''(-1)$.
b. For what values of x in the open interval (-2,2) is g increasing? Explain your reasoning.
c. For what values of x in the open interval (-2, 2) is the graph of g concave down? Explain your reasoning.
d. Sketch the graph of g on the closed interval [-2, 2].



215. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(NOTE: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

a. Find the volume V of water in the container when h = 5 cm. Indicate units of measure.
b. Find the rate of change of the volume of water in the container, with respect to time, when h = 5 cm. Indicate units of measure.
c. Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

216. Let *f* be a function that is differentiable for all real numbers. The table above gives the values of *f* and the derivative *f* ' for selected points x in the closed interval $-1.5 \leqslant x \leqslant 1.5$. The second derivative of *f* has the property that $f''(x) > 0$ for $-1.5 \leqslant x \leqslant 1.5$.

a. Evaluate $\int_0^{1.5} (3f'(x)+4)dx$. Show the work that leads to your answer.

b. Write an equation of the line tangent to the graph of *f* at the point where x = 1. Use this line to approximate the value of *f*(1.2). Is this approximation greater than or less than the actual value of *f*(1.2)? Give a reason for your answer.

c. Find a positive real number r having the property that there must exist a value c with $0 < x < 0.5$ and $f''(c) = r$. Give a reason for your answer.

d. Let *g* be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & for\ x < 0. \\ 2x^2 + x - 7 & for\ x \geq 0. \end{cases}$

The graph of *g* passes through each of the points (x, *f(x)*) given in the table above. Is it possible that *f* and *g* are the same function? Give a reason for your answer.